CHAPTER 4

Work, Energy and Power

Topic Covered

- Notions of work and kinetic energy
- The work-energy theorem
- Work
- Kinetic energy
- Work done by a variable force
- The work-energy theorem for a variable force
- The concept of potential energy
- The conservation of mechanical energy
- The potential energy of a spring
- Various forms of energy
- The law of conservation of energy
- Power
- Collisions

INTRODUCTION

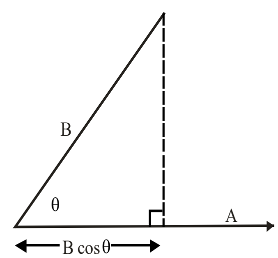
'Work', 'Energy' and 'Power' are the terms which we frequently use in everyday language. Woman carrying water from a well to her house is said to be working. In a drought affected region she may be required to carry it over large distances. If she can do so, she is said to have a large stamina or energy. Energy is thus the capacity to do work. The term power is usually associated with speed. In karate, a powerful punch is one delivered at great speed. This shade of meaning is close to the meaning of the word 'power' used in physics. We shall find that there is at best a loose correlation between the physical definitions and the physiological pictures these terms generate in our minds. The aim of this chapter is to develop an understanding of these three physical quantities.

Before we proceed to this task, we need to develop a mathematical prerequisite, namely the scalar product of two vectors.

SCALAR PRODUCT (DOT PRODUCT)

The scalar product or dot product of any two vectors **A** and **B**, denoted as **A.B** (read **A** dot **B**) is defined as

$$\vec{A}.\vec{B} = A\,B\cos\theta$$



Where θ is the angle between the two vectors Each vector, **A** and **B**, has a direction but their scalar product does not have a direction.

NOTIONS OF WORK AND KINETIC ENERGY

The Work-Energy Theorem

The following relation for rectilinear motion under constant acceleration **a** has been encountered in Chapter **Motion.**

$$v^2 - u^2 = 2as \qquad ...(i)$$

Where **u** and **v** are the initial and final speeds and **s** the distance traversed.

Multiplying both sides by m/2, we have

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2}m(2as) = m\,a\,s \qquad ...(ii)$$

Where the last step follows from Newton's Second Law.

The above equation (ii) provides a motivation for the definitions of work and kinetic energy. A force on the body does work over a certain displacement.

It is also a special case of the work-energy (WE) theorem: $K_f - K_i = W = F.S.$

The change in kinetic energy of a particle is equal to the work done on it by the net force.

For translatory motion : Work done by all the external forces acting on a body is equal to change in its kinetic energy of translation.

Work done by all the external forces = $W = \text{Change in K.E.} = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = \Delta K.E$

For rotational motion : Work done by all the external torque acting on a rigid body is equal to change in its rotational kinetic energy. Work done by all the external torque = $\frac{1}{2}I\omega_1^2 - \frac{1}{2}I\omega_2^2$

Note : In simple words $\Delta K = K_f - K_i = W$ in the work energy theorem if only energy changed is kinetic energy.

WORK

- Whenever a force acting on a body displaces it, work is said to be done by the force.
- Work done by a force is equal to scalar product of force applied and displacement of the body.

Force — Constant
Force — Variable force

DEFINITION

The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement. Thus

$$W = (F\cos\theta)\, d = \vec{F}, \vec{d}$$

Work done by a constant force

If the direction and magnitude of a force applying on a body is constant, the force is said to be constant. Work done by a constant force,

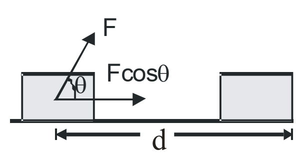
W = Force × component of displacement along force = displacement × component of force along displacement.

If a $\vec{F}$ force is acting on a body at an angle θ to the horizontal and the displacement $\vec{d}$ is along the horizontal, the work done will, be

$W = (F\cos\theta)\, d$

$= F\,(d\cos\theta)$

In vector from, $W = \vec{F}.\vec{d}$



If $\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$ and $\vec{d} = \hat{i}x + \hat{j}y + \hat{k}z$, the work done will be, $W = F_x x + F_y y + F_z z$

Note : The force of gravity is the example of constant force; hence work done by it is the example of work done by a constant force.

UNIT AND SYMBOL

Alternative Units of Work/Energy in J

1 erg = 10^{-7} J

1 Electron volt (eV) = 1.6×10^{-19} J

1 calorie   4.186 J

1 Kilowatt hour (kWh) = 3.6×10^{6} J

Example :

A cyclist comes to a skidding stop in 20 m. During this process, the force on the cycle due to the road is 500 N and is directly opposed to the motion. (A) How much work does the road do on the cycle?

Solution :

Work done on the cycle by the road is the work done by the stopping (frictional) force on the cycle due to the road.

(A) The stopping force and the displacement make an angle of 180^0 (π rad) with each other.

Thus, work done by the road,

$W = F.d \cos\theta$

$= 500 \times 20 \times \cos \pi$

$= -10000$ J

It is the negative work that brings the cycle to a halt in accordance with WE theorem.

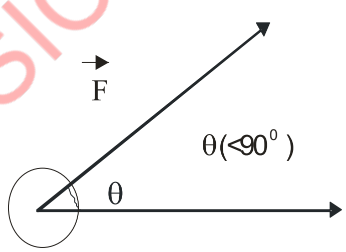
Nature of Work

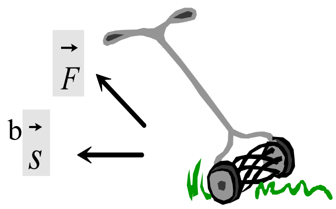
Although work done is a scalar quantity, its value may be positive, negative or even zero.

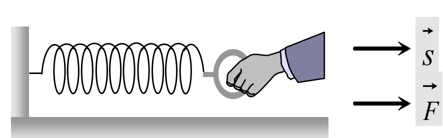
Positive work: - When Q is obtuse ($<90^0$), cos θ is positive. Hence work done is positive.

As $W = \vec{F}.\vec{d} = F\, d \cos\theta$

When θ is acute ($<90^o$) cos θ is positive. Hence work done is positive.





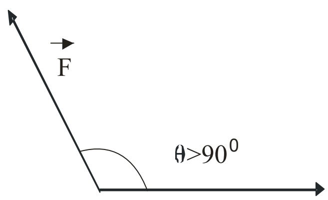


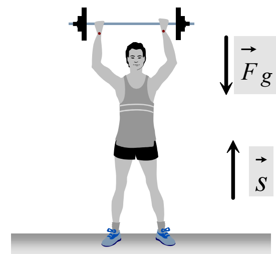
Example :

(i) When a body falls freely under the action of gravity $\theta = 0^o$, $\cos\theta = +1$, therefore work done by gravity on a body, falling freely is positive.

Negative Work: - When θ is obtuse ($>90^0$), $\cos\theta$ is negative. Hence work done is negative.

When θ is obtuse ($>90^o$), $\cos\theta$ is negative. Hence work done is coregative



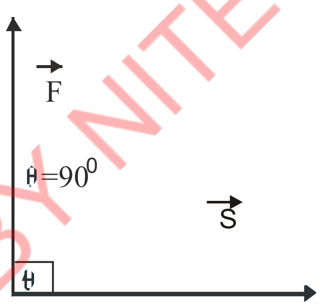


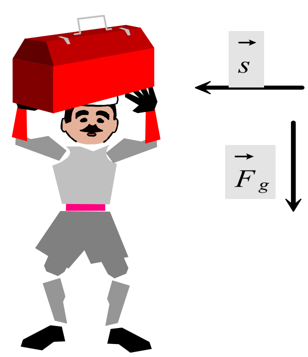
(ii) When a body is thrown up, its motion is opposed by gravity. The angle θ between the gravitational force $\vec{F}$ and displacement $\vec{d}$ is 180º. As $\cos\theta = -1$, therefore, work done by gravity is negative.

(iii) When a body is moved over a rough horizontal surface, the motion is opposed by the force of friction. Hence work done by frictional force in negative. **Note that work done by the applied force is not negative**

(iv) When a positive charge is moved closer to another positive charge, work done by electrostatic force of repulsion between the charges is negative.

Zero Work: - When force applied F or the displacement s or both are zero, work done W= F.d cos θ is zero. Again, when angle θ between F and d is 90^0. Therefore, work done is zero.

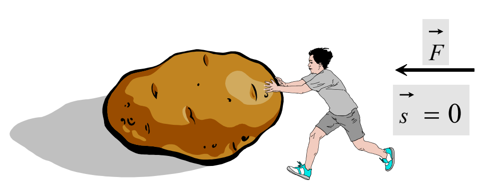




Example :

(i) When we fail to move a heavy stone, however hard we may try, work done by us is zero,

$\vec{d} = 0$



(ii) When a collie carrying some load on his head moves on horizontal platform, $\theta = 90^o$. Therefore, workdone by the collie is zero. This is because $\theta = 90^o$

(iii) Tension in the string of simple pendulum is always perpendicular to displacement of the bob. Therefore, work done by tension is always zero.

Note : Another way of expressing negative or positive work is that when energy is transferred to the object work done is positive and when energy is transferred from object the work done is negative and hence the work which is a transfer of energy has same dimensions as energy.

SOLVED EXAMPLE

Example 1. A body of mass 5 *kg* is placed at the origin, and can move only on the *x*-axis. A force of 10 *N* is acting on it in a direction making an angle of 60^o with the *x*-axis and displaces it along the *x*-axis by 4 *metres*. The work done by the force is

(A) 2.5 J (B) 7.25 J

(C) 40 J (D) 20 J

Solution : **(D)**

Work done $= \vec{F}.\vec{s} = Fs\cos\theta = 10\times4\times\cos60° = 20J$

Example 2. A force $F = (5\hat{i} + 3\hat{j})N$ is applied over a particle which displaces it from its origin to the point $r = (2\hat{i} - 1\hat{j})$ metres. The work done on the particle is

(A) –7 J (B) +13 J

(C) +7 J (D) +11 J

Solution : **(C)**

Work done $= \vec{F}.\vec{r} = (5\hat{i} + 3\hat{j}).(2\hat{i} - \hat{j}) = 10 - 3 = +7J$

Example 3. A horizontal force of 5 N is required to maintain a velocity of 2 m/s for a block of 10 kg mass sliding over a rough surface. The work done by this force in one minute is

(A) 600 J (B) 60 J

(C) 6 J (D) 6000 J

Solution : **(A)**

Work done = Force × displacement = F × s = F × v × t = 5 × 2 × 60 = 600 J.

EXERCISE

1. A box of mass 1 kg is pulled on a horizontal plane of length 1 m by a force of 8 N then it is raised vertically to a height of 2m, the net work done is

(A) 28 J (B) 8 J

(C) 18 J (D) None of above

2. A 10 kg satellite completes one revolution around the earth at a height of 100 km in 108 minutes. The work done by the gravitational force of earth will be

(A) $108 \times 100 \times 10\ J$

(B) $\frac{108 \times 10}{100} J$

(C) $\frac{100 \times 10}{108} J$

(D) Zero

ENERGY

The energy of a body is defined as the capacity of doing work. It is measured by the total amount of workthat a body can do.

Energy is a scalar quantity

Unit : Its unit is same as that of work or torque.

In MKS : Joule, watt sec

In CGS : Erg

Note : 1 eV = 1.6×10^{-19} joule

1 KWh = 36×10^5 joule

10^7 erg = 1 joule

Dimension $[M^1L^2T^{-2}]$

According to Einstein's mass energy equivalence principle mass and energy are inter convertible i.e. they can be changed into each other

Energy equivalent of mass m is, $E = mc^2$

Where, m : mass of the particle

c : velocity of light

E : equivalent energy corresponding to mass m.

In mechanics we are concerned with mechanical energy only which is of two type

(A) kinetic energy (ii) potential energy

KINETIC ENERGY

The energy possessed by a body by virtue of its motion is called kinetic energy. Expression for K.E.: Consider a body of mass m, initially at rest . Let a constant force (F) be acting on the body, due to which it starts moving with acceleration a.

Then F = ma ...(i)

Let it acquires velocity v after travelling distance d, then $v^2 - u^2 = 2ad$, we have $v^2 = 2ad$

$$\Rightarrow a = \frac{v^2}{2d} \quad ...(ii)$$

from (i)& (ii) we get $F = \frac{mv^2}{2d}$

As $W = \frac{mv^2}{2d} x d = \frac{mv^2}{2} = \frac{1}{2} mv^2$

If a body of mass m is moving with velocity v, its kinetic energy

$KE = \frac{1}{2}mv^2$, for translatory motion

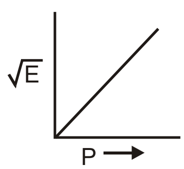
$KE = \frac{1}{2}I\omega^2$, for rotational motion

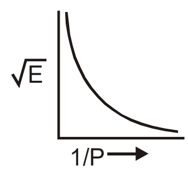
Kinetic energy is always positive

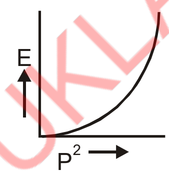
If linear momentum of body is p, $K.E. = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$ - for translatory motion

If angular momentum of body is J, $KE = \frac{J^2}{2I} = \frac{1}{2}I\omega^2$ - for rotational motion

p or J $\alpha \sqrt{E}$ p : momentum E : kinetic energy







The kinetic energy of a moving body is measured by the amount of work which has been done in bringing the body from the rest position to its present moving position or

The kinetic energy of a moving body is measured by the amount of work which the body can do against the external forces before it comes to rest.

If a body performs translatory and rotational motion simultaneously, its total kinetic energy = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

DEFINITION OF ELECTRON- VOLT

The energy acquired by an electron when it passes through a potential difference of 1 volt is called electron-volt.

$1\ eV = 1.6 \times 10^{-19} C \times 1\ volt$

SOLVED EXAMPLE

Example 1. Consider the following two statements

1. Linear momentum of a system of particles is zero
2. Kinetic energy of a system of particles is zero

Then

(A) 1 implies 2 and 2 implies 1 (B) 1 does not imply 2 and 2 does not imply 1

(C) 1 implies 2 but 2 does not imply 1 (D) 1 does not imply 2 but 2 implies 1

Solution : **(D)**

Momentum is a vector quantity whereas kinetic energy is a scalar quantity. If the kinetic energy of a system is zero then linear momentum definitely will be zero but if the momentum of a system is zero then kinetic energy may or may not be zero.

Example 2. A running man has half the kinetic energy of that of a boy of half of his mass. The man speeds up by 1 *m/s* so as to have same *K.E.* as that of boy. The original speed of the man will be

(A) $\sqrt{2}\,m/s$ (B) $(\sqrt{2}-1)\,m/s$

(C) $\frac{1}{(\sqrt{2}-1)}\,m/s$ (D) $\frac{1}{\sqrt{2}}\,m/s$

Solution : **(C)**

Let m = mass of the boy, M = mass of the man, v = velocity of the boy and V = velocity of the man

Initial kinetic energy of man $= \frac{1}{2}MV^2 = \frac{1}{2}\left[\frac{1}{2}mv^2\right] = \frac{1}{2}\left[\frac{1}{2}\left(\frac{M}{2}\right)v^2\right]$ $\left[\because\ m = \frac{M}{2}\right]$

$$V^2 = \frac{v^2}{4} \Rightarrow V = \frac{v}{2} \qquad \text{...(i)}$$

When the man speeds up by 1 *m/s*, $\frac{1}{2}M(V+1)^2 = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{M}{2}\right)v^2 \Rightarrow (V+1)^2 = \frac{v^2}{2}$

$$\Rightarrow V + 1 = \frac{v}{\sqrt{2}} \qquad \text{...(ii)}$$

From (i) and (ii) we get speed of the man $V = \left(\frac{1}{\sqrt{2}-1}\right)m/s$.

Example 3 A body of mass 10 *kg* at rest is acted upon simultaneously by two forces 4*N* and 3*N* at right angles to each other. The kinetic energy of the body at the end of 10 sec is

(A) 100 *J* (B) 300 *J*

(C) 50 *J* (D) 125 *J*

Solution : **(D)**

As the forces are working at right angle to each other therefore net force on the body

$$F = \sqrt{4^2 + 3^2} = 5N$$

Kinetic energy of the body = work done = $F \times s$

$$= F \times \frac{1}{2}at^2 = F \times \frac{1}{2}\left(\frac{F}{m}\right)t^2 = 5 \times \frac{1}{2}\left(\frac{5}{10}\right)(10)^2 = 125\ J.$$

Example 4. If the momentum of a body increases by 0.01%, its kinetic energy will increase by

(A) 0.01% (B) 0.02 %

(C) 0.04 % (D) 0.08 %

Solution : **(B)**

Kinetic energy $E = \frac{p^2}{2m} \therefore E \propto p^2$

Percentage increase in kinetic energy = 2(% increase in momentum)

[If change is very small]

= 2(0.01%) = 0.02%.

EXERCISE

1. If the momentum of a body is increased by 100 %, then the percentage increase in the kinetic energy is

(A) 150 % (B) 200 %

(C) 225 % (D) 300 %

2. A body of mass 5 *kg* is moving with a momentum of 10 *kg-m/s*. A force of 0.2 *N* acts on it in the direction of motion of the body for 10 seconds. The increase in its kinetic energy is

(A) 2.8 *J* (B) 3.2 *J*

(C) 3.8 *J* (D) 4.4 *J*

3. Two masses of 1*g* and 9*g* are moving with equal kinetic energies. The ratio of the magnitudes of their respective linear momenta is

(A) 1 : 9 (B) 9 : 1

(C) 1 : 3 (D) 3 : 1

4. A body of mass 2 *kg* is thrown upward with an energy 490 *J*. The height at which its kinetic energy would become half of its initial kinetic energy will be-

(A) 35 *m* (B) 25 *m*

(C) 12.5 *m* (D) 10 *m*

5. A 300 *g* mass has a velocity of *m/sec* at a certain instant. What is its kinetic energy

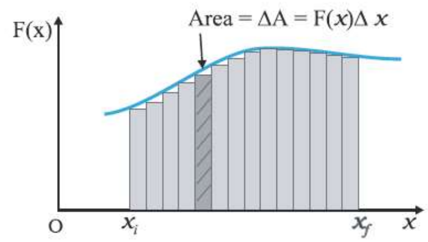
(A) 1.35 *J* (B) 2.4 *J*

(C) 3.75 *J* (D) 7.35 *J*

WORK DONE BY A VARIABLE FORCE

A constant force is rare. It is the variable force, which is more commonly encountered. Given fig. is a plot of a varying force in one dimension. If the displacement Δx is small, we can take the force F (x) as approximately constant and the work done is then

W =F (x) Δ x

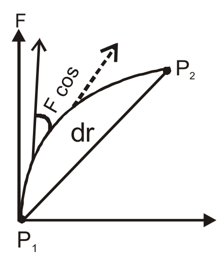


This is illustrated in Fig. Adding successive rectangular areas in Fig. 6.3(A) we get the total work done as

$$W \cong \sum_{x_t}^{x_f} F(x).\Delta x$$

If the force applying on a body is changing in its direction or magnitude or both, the force is said to be variable suppose a constant force causes displacement in a body from position P_1 to position P_2. To calculate the work done by the force the path from P_1 to P_2 can be divided into infinitesimal element, each element is so small that during displacement of body through it, the force is supposed to be constant. It $d\vec{r}$ be small displacement of body and $\vec{F}$ be the force applying on the body, the work done by force is dW = $\vec{F}.d\vec{r}$ ......

(i) The total work done in displacing body from P_1 to P_2 is given



by, $\int dW = \int_{P_1}^{P_2} \vec{F}.d\vec{r}$

or $W = \int_{P_1}^{P_2} \vec{F}.d\vec{r}$

If $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the points P_1 and P_2 respectively, the total work done will be -

$$W = \int_{r_1}^{r_2} \vec{F}.dr$$

Note : When we consider a block attached to a spring, the force on the block is k times the elongation of the spring, where k is spring constant. As the elongation changes with the motion of the block, therefore the force is variable. This is an example of work done by variable force.

Work-Energy Theorem for Variable Force

We know, kinetic energy, $K.E. = \frac{1}{2}mv^2$

Differentiating both sides w.r.t 't', we get

$$\frac{d(K.E.)}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt}$$

But $\frac{dv}{dt} = a \therefore \frac{d(K.E.)}{dt} = mav$

or $\quad \frac{d(K.E.)}{dt} = Fv \quad$ ($\because$ F = ma, Newton's second law of motion.)

Since $v = \frac{dx}{dt} \quad \frac{d(K.E.)}{dt} = F\frac{dx}{dt}$ or $d(K.E.) = Fdx \quad$ ...(1)

When $x = x_i$, K.E. = $(K.E.)_i$ and when $x = x_f$, K.E. = $(K.E.)_f$.

Integrating equation (1) between these limits, we get

$$\int_{(K.E.)_i}^{(K.E.)_f} d(K.E.) = \int_{x_i}^{x_f} Fdx \text{ or } \int_{x_i}^{x_f} Fdx = [K.E.]_{(K.E.)_i}^{(K.E.)_f} = (K.E.)_f - (K.E.)_i$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

But $\quad \int_{x_i}^{x_f} Fdx$ = Work done (W) by the variable force.

$$\therefore \quad W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \text{change in K.E.}$$

This is work-energy theorem.

Discussion of Work-Energy Theorem

(i) Work done by a force is zero, if there is no change in the speed of a particle or a body.

Explanation: $W - \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

If u = v, we get

Example :

When a particle moves in a circular path with constant speed, then work done by the centripetal force is zero. In this case, there is no change in the kinetic energy of the particle and hence according to work-energy theorem, work done is zero.

(ii) Work done by a force is negative, if there is decrease in the kinetic energy of a particle or a body.

Solution : $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Since $\quad \frac{1}{2}mv^2 < \frac{1}{2}mu^2$

$\therefore \quad$ W = negative

Example :

When a particle is projected upward, then the work done by the gravitational force is negative. In this case, as the particle moves up, its speed decreases and hence consequently the kinetic energy of the particle also decreases.

(iii) Worked done by the force is positive, if there is increase in the velocity and hence kinetic energy of the particle.

Explanation : $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Since $\frac{1}{2}mv^2 > \frac{1}{2}mu^2$

$\therefore$ W = Positive.

Example :

When a particle or an object is dropped from the top of a building, then the work done by the gravitational force is positive. As the particle descends, its speed increases and hence K.E. of the particle also increases. This implies that the work done by the gravitational force is positive.

SOLVED EXAMPLE

Example 1. A force F = (10 + 0.50x) acts on a particle in X direction, where F is in Newton and X in metre. Find the work done by this force during a displacement from X = 0 to X = 5 m.

Solution: Here, F = (10 + 0.50x)

Small amount of work done in moving the particle through a small distance dx is

$$dW = \vec{F}.\vec{d}x = (10 + 0.50x)dx$$

Total work done,

x = 5

$$W = \int_{x=0}^{x=5} (10 + 0.50x)dx$$

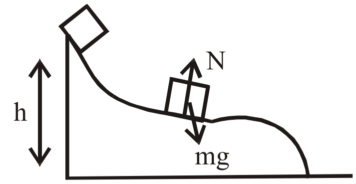
$$W = \left[10x + 0.5\frac{x^2}{2}\right]_0^5 = 10(5-0) + \frac{0.5}{2}(5^2 - 0) = 50 + 6.25 = 56.25 \text{ Joule.}$$

Example 2. A particle moves along some trajectory X-Y plane from point P whose position vector is $\vec{r}_1 = \left(\vec{i} + 2\vec{j}\right)$m to point Q, whose position vector is $\vec{r}_2 = \left(2\vec{i} - 3\vec{j}\right)$m. During motion it experienced the action of certain forces one of which is $F = \left(3\vec{i} + 4\vec{j}\right)$N. Find the work performed by $\vec{F}$

Solution: Since $\vec{F}$ is constant, work done by it is simply given by

$$W = \vec{F}.\vec{S} = \vec{F}.\left(\vec{r}_2 - \vec{r}_1\right) = \left(3\vec{i} + 4\vec{j}\right).\left(\vec{i} - 5\vec{j}\right) = 3 - 20 = -17 \text{ Joules}$$

Example 3. A body of mass m slides down the smooth surface of a hill of height h. Find the velocity of the block at the bottom of the hill (See figure)



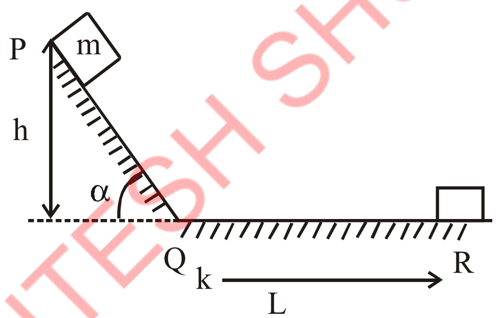
Solution. There are only two forces namely N and mg that act on the body. The work done by $\vec{N}$ is zero since it is always perpendicular to velocity $\vec{v}$, i.e.

$$W_N = \int \vec{N}.\vec{ds} = \int \vec{N}.\vec{v}\,dt = 0$$

The work done by gravity = mgh.
Let the velocity of the block at the bottom be v. From work - energy theorem, we have

$$\frac{1}{2}mv^2 - 0 = mgh + 0 \Rightarrow v = \sqrt{2gh}.$$

Example 4 A small block of mass m slides down an inclined plane at an angle α to the horizontal. Find the distance that it will move on the horizontal plane after sliding down the incline. The height of the inclined plane is h and the coefficient of kinetic friction over both surfaces is μ_k



Solution. Let us choose P and R as initial and final positions. The velocity of block at these points is zero. Thus,

$$\Delta kE = 0$$

$$W_{gravity} = mgh\ ,\ W_{friction} = W_{P\to Q} + W_{Q\to R}$$

$$wfr = -\mu_k mg\cos\alpha \times \frac{h}{\sin\alpha} + -\mu_k mgL\ ,\ W_{Normal\ reaction} = 0$$

Using work theory theorem, we find

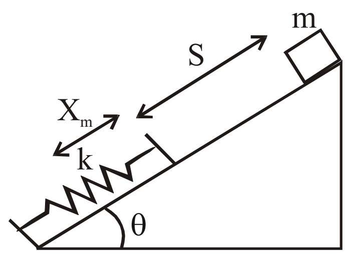
Δ K.E. = Work by all

Δ K.E. = $w_{mg} + W_N + W_{fr}$

$$0 = mgh + 0 - \mu_k mg\cos\alpha \times \frac{h}{\sin\alpha} - \mu_k mgL$$

$$\Rightarrow L = h\left(\frac{1}{\mu_k} - \cot\alpha\right)$$

Example 5. An ideal massless spring with spring constant k is placed at the bottom of a frictionless inclined plane which makes an angle θ with the horizontal (See fig.). A block of mass M is released from rest at the top of the incline. It moves a distance S along the incline before it touches the spring. Find (A) the maximum compression of the spring and (B) the maximum speed of the block.



Solution. **(A)** Let the maximum compression of the spring be X_m at this position, the block will come to instantaneous rest.

$$W_{spring} = \frac{1}{2}k\left(x_f^2 - x_i^2\right) = -\frac{1}{2}k\,x_m^2 \qquad W_{gravity} = mg\left(S + x_m\right)\sin\theta$$

The initial and final velocity of the block is zero.

Thus $\Delta kE = 0$

From work - energy theorem, we have $0 = -\frac{1}{2}k\,x_m^2 + mg\left(S + x_m\right)\sin\theta$

$$\Rightarrow \frac{1}{2}k\,x_m^2 - mg\sin\theta\,x_m - mgS\sin\theta = 0$$

$$\Rightarrow x_m = \frac{mg\sin\theta + \sqrt{m^2g^2\sin^2\theta + 2kmgS\sin\theta}}{K}$$

(The other value of X_m is not possible)

(B) The velocity of the block will continue to increase as long as its acceleration a is down the incline. From Newton's second law, we have

$$mg\sin\theta - kx = ma$$

$$\Rightarrow a = \frac{mg\sin\theta - kx}{m}$$

It becomes zero, when $mg\sin\theta - kx_0 = 0$

$$\Rightarrow x_0 = \frac{mg\sin\theta}{K}$$

Let the velocity of the block be V_m at this position. From work - energy theorem,we have

$$\frac{1}{2}mv_m^2 - 0 = mg(S+x_o)\sin\theta - \frac{1}{2}kx_0^2 \qquad = mgS\sin\theta + \frac{(mg\sin\theta)^2}{2K}$$

$$\Rightarrow V_m = \sqrt{mg\sin\theta\frac{(2kS+mg\sin\theta)}{K}}$$

Example 6. A 1,00,000 kg engine is moving up a slope of gradient 5° at a speed of 100 meter / hour. The coefficient of friction between the engine and rails is 0.1. If the engine has an efficiency of 4% for converting heat into work, find the amount of coal the engine has to burn up in one hour (Burning of 1 kg of coal yields 50,000 joule)

Solution Work done by friction in one hour, $w_1 = -(\mu mg\cos\theta)\mu \; = -mg\cos\theta v$

Work done by gravity in one hour, $w_2 = -mgv\sin\theta$

Work done by engine, $w_3 = ?$

Since the speed of the engine is constant, work energy theorem yields

$$0 = w_1 + w_2 + w_3 \qquad \Rightarrow w_3 = -w_1 - w_2$$

$$= mg[\sin\theta + \mu\cos\theta]v$$

$$= 10^5 \times 9.8[\sin 5^\circ + 0.1\cos 5^\circ] \times 100$$

$$= 1.83 \times 10^7 \text{ Joule}$$

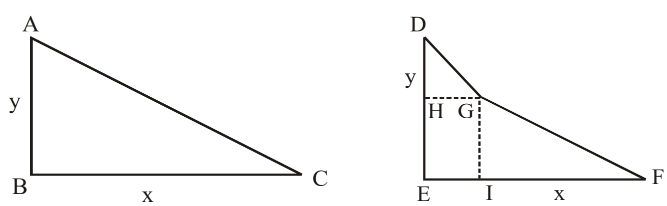
Let H be the amount of heat needed. Then

$$\frac{4}{100}H = 1.83 \times 10^7 \text{ J} \qquad \Rightarrow H = 45.75 \times 10^7 \text{ J}$$

The amount of coal need in one hour

$$= \frac{45.75 \times 10^7}{5 \times 10^4} = 9.15 \times 10^3 \text{ kg}$$

Example 7 In the figure shown below, AC, DG and GE are fixed inclined planes. BC = EF = x and AB = DE = y. A small block of mass M is released from the point A. It slides down AC and reaches C with a speed Vc. The same block is released from rest from point D. It slides down DGF and reaches the point F with speed V_F. The coefficient of kinetic friction between the block and both surfaces AC and GDF is μ Calculate (A) V_c and (B) V_F.



Solution. (A) Let angle $ABC = \theta$, then

$$AC = \frac{y}{\sin\theta} = \frac{x}{\cos\theta}$$

Work done by force of friction $= -\mu mg\cos\theta \times AC$

Work done by gravity = mg y

From work - energy theorem,

$$\Delta K = -\mu mg\cos\theta AC + mgy$$

$$\Rightarrow \frac{1}{2}mV_c^2 - 0 = mg(y - \mu x)$$

$$\Rightarrow V_c = \sqrt{2g(y - \mu x)}$$

(B) Let angle GFI = α and angle DGH = β, then

$$DG = \frac{HG}{\cos\beta} = \frac{EI}{\cos\beta}$$

$$GF = \frac{FI}{\cos\alpha}$$

Work done by friction along with path $D \to G \to F$

$$= -\mu mg\cos\beta \times DG - \mu mg\cos\alpha\ GF$$

$$= -\mu mg\ EI - \mu mg\ IF$$

$$= -\mu mgx$$

Work done by gravity = mg DE = mg y

From work - energy theorem, we get

$$\Delta kE = -\mu mgx + mgy$$

$$\Rightarrow \frac{1}{2}mV_F^2 - 0 = mg(y - \mu x)$$

$$\Rightarrow V_F = \sqrt{2g(y - \mu x)}$$

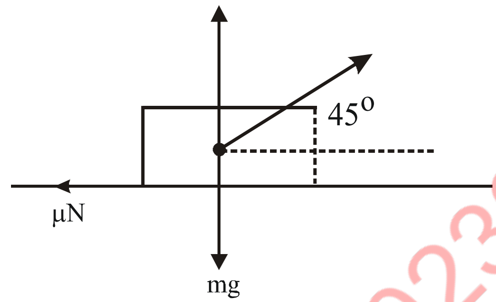
Example 8. A man pulls a 5 kg block 20 meters along a horizontal surface at a constant speed with force directed 45^o above the horizontal. If the coefficient of kinetic friction is 0.2, how much work does the man do on the block?

Solution. The f.b.d. of the block is shown below.

As the block moves with constant velocity, force on it must balance. Hence

$$N - mg + F\sin 45^{\circ} = 0 \qquad ...(1)$$

$$F\cos 45^{\circ} - \mu N = 0 \qquad ...(2)$$



From equation (1) $N = mg - \dfrac{F}{\sqrt{2}}$

Putting this value in equation (2), we get

$$\frac{F}{\sqrt{2}} = \mu\left(mg - \frac{F}{\sqrt{2}}\right)$$

$$\Rightarrow \; F = \frac{\sqrt{2}\mu mg}{1.2} = 11.55 \text{ Newton}$$

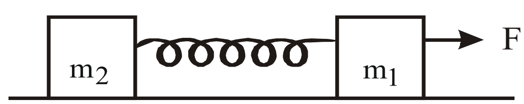
Work done by the man = $F\cos 45^{o} \times d$

$$= \frac{11.55}{\sqrt{2}} \times 20 = 162.32\text{J}$$

Example 9. Two blocks of masses m_1 and m_2 are connected by spring of spring constant K. Initially the spring is at its natural length. The coefficient of friction between the bars and the surface is μ. What minimum constant force has to be applied in the horizontal direction to the block of mass m, in order to shift the other block?

Solution. Block m_2 will move when spring pulls it to right by a force equal to force of friction

i.e. $\mu_1 m_2 g = kx$



$$\Rightarrow x = \frac{\mu m_2 g}{K}$$

Therefore, block m_1 must move a distance equal to $\dfrac{\mu m_2 g}{K}$ before it comes to rest.

Applying work - energy theorem to block m_1, we have

Change in KE = $W_F + W_{friction} + W_{spring}$

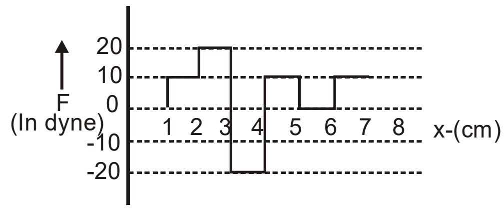
$$\Rightarrow \; 0 = F\frac{\mu m_2 g}{K} - \mu m_1 g\frac{\mu m_2 g}{2k} - \frac{1}{2}K\left(\frac{\mu m_2 g}{K}\right)^2$$

$$\Rightarrow F = \mu\left(m_1 + \frac{m_2}{2}\right)g$$

EXERCISE

1. A position dependent force $\vec{F} = 7 - 2x + 3x^2$ acts on a small body of mass 2kg and displaces it from x = 0 to x = 5 m. The work done in joule will be

2. For the force displacement diagram shown in adjoining diagram the work done by the force in displacing the body from x = 1 cm to x = 5 cm is -

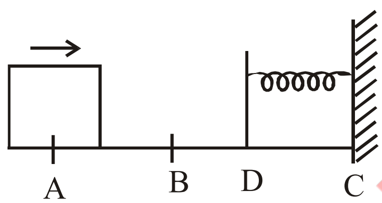


3. A uniform chain of mass M and length L is lying on a frictionless table in such a way that its 1/3 part is hanging vertically down. The work done in pulling the chain up the table is

4. The work done in pulling a body of mass 5 kg along an inclined plane (angle 60º) with coefficient of friction 0.2 through 2m, will be

5. A force $\vec{F} = (7 - 2x + 3x^2)N$ is applied on a 2 kg mass which displaces it from x = 0 to x = 5 m. What is the work done in joule ?

6.



As 0.5 kg body slides from the point A (see fig) on a horizontal track with an initial speed of 3m/s towards a massless horizontal spring of length 1m and force constant 2N/m. The part AB of the track is frictionless and part BC has coefficient of static and dynamic friction 0.22 and 0.2 respectively. Find the total distance through which the block moves before it comes to rest completely $\left(g = 10m/s^2\right)$ if $AB = 2m$ and $BD = 2.14m$

7. A bullet fired with a velocity v_o penetrates a plank and looses one third of its velocity. It then strikes another plank which it just penetrates through. Find the ratio of the thickness of the planks supposing average resistance to the penetration is same in both the planks.

8. A position dependent force $\vec{F} = (7 - 2x + 3x^2)N$ acts on a small abject of mass 2 kg to displace it from x = 0 to x = 5m. The work done in joule is

(A) 70 J (B) 270 J

(C) 35 J (D) 135 J

9. A particle moves under the effect of a force $F = Cx$ from $x = 0$ to $x = x_1$. The work done in the process is

(A) Cx_1^2 (B) $\frac{1}{2}Cx_1^2$

(C) Cx_1 (D) Zero

10. The vessels A and B of equal volume and weight are immersed in water to a depth h. The vessel A has an opening at the bottom through which water can enter. If the work done in immersing A and B are W_A and W_B respectively, then

(A) $W_A = W_B$ (B) $W_A < W_B$

(C) $W_A > W_B$ (D) $W_A >=< W_B$

11. Work done in time t on a body of mass m which is accelerated from rest to a speed v in time t_1 as a function of time t is given by

(A) $\frac{1}{2} m \frac{v}{t_1} t^2$ (B) $m \frac{v}{t_1} t^2$

(C) $\frac{1}{2}\left(\frac{mv}{t_1}\right)^2 t^2$ (D) $\frac{1}{2} m \frac{v^2}{t_1^2} t^2$

POTENTIAL ENERGY

Definition: - The potential energy of a body is defined as the energy possessed by the body by virtue of its position or configuration in some field. It is of two type -

(A) Gravitational potential Energy (B) Elastic Potential Energy

OR

- The energy which a body has by virtue of its position or configuration in a conservative force field
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Potential energy of a body at any position in a conservation force field is defined as the workdone by an external agent against the action of conservation force in order to shift it from reference point.
- Relationship between conservative force field and potential energy (U) $\vec{F} - \nabla U = -\text{grad}\ (U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$

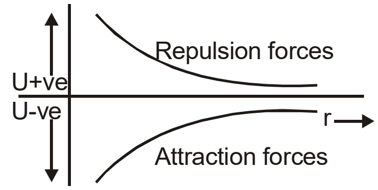
Example. (i) $U = 3x^2 \Rightarrow \vec{F} = -6x\hat{i}$ (ii) $U = 2x^2y + 3y^2x + xz^2$

$\Rightarrow \vec{F} = -(4xy + 3y^2 + z^2)\hat{i} + (2x^2 + 6xy)\hat{j} + (2xz)\hat{k}$

- If force varies only with one dimension then $F = -\frac{dU}{dx}$ or $U = -\int_{x_1}^{x_2} F dx$
- Potential energy may be positive or negative

  (i) Potential energy is positive, if force field is repulsive in nature

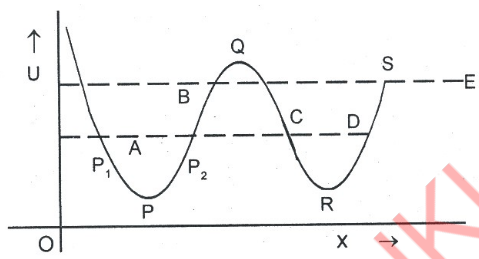
  (ii) Potential energy is negative, if force field is attractive in nature



- If r ↑ (separation between body and force centre), U ↑, force field is attractive or vice-versa.
- If r ↑, U ↓, force field is repulsive in nature.

Potential energy curve

A graph plotted between the PE of a particle and its displacement from the centre of force field is called PE curve



Using graph, we can predict the rate of motion of a particle at various positions.

Force on the particle is $F_{(x)} = -\frac{dU}{dx}$

Case : I On increasing x, if U increase, force is in (–)ve x-direction i.e. attraction force.

Case : II On increasing x, if U decreases, force is in (+)ve x-direction i.e. repulsion force.

Different positions of a particle

Position of equilibrium

If net force acting on a body is zero, it is said to be in equilibrium for equilibrium

$\frac{dU}{dx} = 0$ Points P, Q, R and S are the states of equilibrium positions.

Types of equilibrium : If net force acting on a particle is zero, it is said to be in equilibrium.

For equilibrium $\frac{dU}{dx} = 0$, but the equilibrium of particle can be of three types :

Stable equilibrium: When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

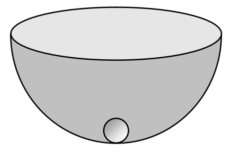
Potential energy is minimum

$$F = -\frac{dU}{dx} = 0$$

$$\frac{d^2U}{dx^2} = \text{positive}$$

i.e. rate of change of $\frac{dU}{dx}$ is positive.

Example : A marble placed at the bottom of a hemispherical bowl.



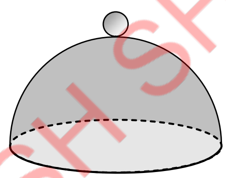
Unstable equilibrium: When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

Potential energy is maximum

$$F = -\frac{dU}{dx} = 0$$

$\frac{d^2U}{dx^2}$ = negative *i.e.* rate of change of $\frac{dU}{dx}$ is negative

Example : A marble balanced on top of a hemispherical bowl



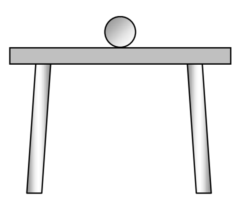
Neutral: When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.

Potential energy is constant.

$$F = -\frac{dU}{dx} = 0$$

$\frac{d^2U}{dx^2} = 0$ *i.e.* rate of change of $\frac{dU}{dx}$ is zero

Example : A marble placed on horizontal table.

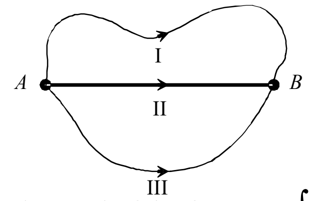


Work Done in Conservative and Non-Conservative Field .

(1) In conservative field work done by the force (line integral of the force *i.e.* $\int \vec{F}.d\vec{l}$) is independent of the path followed between any two points.

$$\underset{\text{Path I}}{W_{A\to B}} = \underset{\text{Path II}}{W_{A\to B}} = \underset{\text{Path III}}{W_{A\to B}}$$

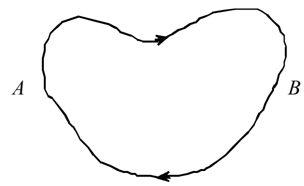
$$\underset{\text{Path I}}{\int \vec{F}.d\vec{l}} = \underset{\text{Path II}}{\int \vec{F}.d\vec{l}} = \underset{\text{Path III}}{\int \vec{F}.d\vec{l}}$$



(2) In conservative field work done by the force (line integral of the force *i.e.* $\int \vec{F}.d\vec{l}$) over a closed path/loop is zero.

$W_{A\to B} + W_{B\to A} = 0$

or $\oint \vec{F}.d\vec{l} = 0$

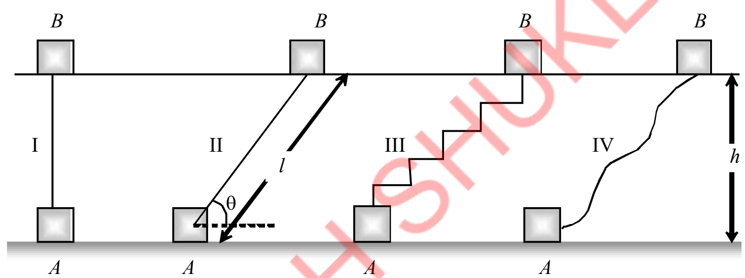


Conservative force : The forces of these type of fields are known as conservative forces.

Example

Electrostatic forces, gravitational forces, elastic forces, magnetic forces *etc* and all the central forces are conservative in nature.

If a body of mass m is lifted to height h from the ground level by different path as shown in the figure



Work done through different paths

$$W_I = F.s = mg \times h = mgh$$

$$W_{II} = F.s = mg\sin\theta \times l = mg\sin\theta \times \frac{h}{\sin\theta} = mgh$$

$$W_{III} = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4 = mg(h_1 + h_2 + h_3 + h_4) = mgh$$

$$W_{IV} = \int \vec{F}.d\vec{s} = mgh$$

It is clear that $W_I = W_{II} = W_{III} = W_{IV} = mgh$.

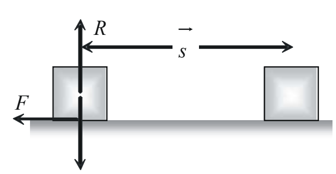
Further if the body is brought back to its initial position A, similar amount of work (energy) is released from the system it means $W_{AB} = mgh$

and $W_{BA} = mgh$.

Hence the net work done against gravity over a round strip is zero.

$W_{Net} = W_{AB} + W_{BA}$

$= mgh + (-mgh) = 0$



i.e. the gravitational force is conservative in nature.

Non-conservative forces : A force is said to be non-conservative if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions and for complete cycle this work done can never be a zero.

Example: Frictional force, Viscous force, Air drag *etc*.

If a body is moved from position *A* to another position *B* on a rough table, work done against frictional force shall depends on the length of the path between *A* and *B* and not only on the position *A* and *B*.

$W_{AB} = -\mu mgs$

Further if the body is brought back to its initial position *A,* work has to be done against the frictional force, which always opposes the motion. Hence the net work done against the friction over a round trip is not zero.

$W_{BA} = -\mu mgs$

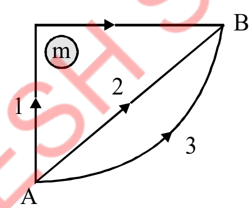
$\therefore \quad W_{Net} = W_{AB} + W_{BA}$

$W_{Net} = -\mu mgs - \mu mgs = -(2\mu mg)s$

i.e. the friction is a non-conservative force.

SOLVED EXAMPLE

Example 1. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation



(A) $W_1 > W_2 > W_3$ (B) $W_1 = W_2 = W_3$

(C) $W_1 < W_2 < W_3$ (D) $W_2 > W_1 > W_3$

Solution : **(B)**

As gravitational field is conservative in nature. So work done in moving a particle from *A* to *B* does not depends upon the path followed by the body. It always remains same.

Example 2. A particle of mass 0.01 *kg* travels along a curve with velocity given by $4\hat{i} + 16\hat{k}$ ms^{-1}. After some time, its velocity becomes $8\hat{i} + 20\hat{j}\ ms^{-1}$ due to the action of a conservative force. The work done on particle during this interval of time is

(A) 0.32 J (B) 6.9 J

(C) 9.6 J (D) 0.96 J

Solution : **(D)**

$v_1 = \sqrt{4^2 + 16^2} = \sqrt{272}$ and $v_2 = \sqrt{8^2 + 20^2} = \sqrt{464}$

Work done = Increase in kinetic energy $= \frac{1}{2}m[v_2^2 - v_1^2] = \frac{1}{2} \times 0.01[464 - 272] = 0.96$ J .

EXERCISE

1. A particle moves under the effect of a force $F = Cx$ from x = 0 to x = x_1. The work done in the process is

(A) Cx_1^2 (B) $\frac{1}{2}Cx_1^2$

(C) Cx_1 (D) Zero

2. The potential energy of a certain spirng when stretched through a distance 'S' is 10 joule. The amount of work (in joule) that must be done on this spring to stretch it through an additional distance 'S'will be

(A) 30 (B) 40

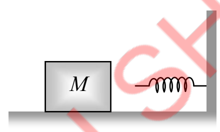
(C) 10 (D) 20

3. The work done by a force $\vec{F} = (-6x^3\hat{i})$ N, in displacing a particle from x = 4m to x = –2 m is

(A) 360 J (B) 240 J

(C) –240 J (D) –360 J

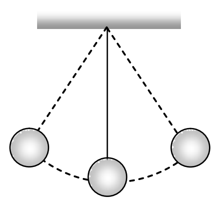
4. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant K and compresses it by length L. The maximum momentum of the block after collision is



(A) Zero (B) $\frac{ML^2}{K}$

(C) $\sqrt{MK}\,L$ (D) $\frac{KL^2}{2M}$

5. What is the velocity of the bob of a simple pendulum at its kilowatt-hour of energy will it deliver to the body as it is digested.



(A) 0.6 m/s (B) 1.4 m/s

(C) 1.8 m/s (D) 2.2 m/s

6. A running man has half the kinetic energy of that of a boy of half of his mass. The man speeds up by 1*m*/*s* so as to have same *K.E.* as that of the boy. The original speed of the man will be

(A) $\sqrt{2}\,m/s$ (B) $(\sqrt{2}-1)m/s$

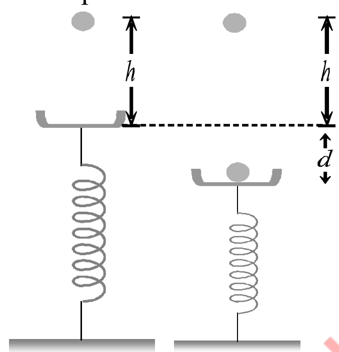
(C) $\frac{1}{(\sqrt{2}-1)}m/s$ (D) $\frac{1}{\sqrt{2}}m/s$

7. A particle of mass m at rest is acted upon by a force F for a time t. Its kinetic energy after an interval t is

(A) $\frac{F^2t^2}{m}$ (C) $\frac{F^2t^2}{2m}$

(D) $\frac{F^2t^2}{3m}$ (E) $\frac{Ft}{2m}$

8. A vertical spring with force constant K is fixed on a table. A ball of mass m at a height h above the free upper end of the spring falls vertically on the spring so that the spring is compressed by a distance d. The net work done in the procees is

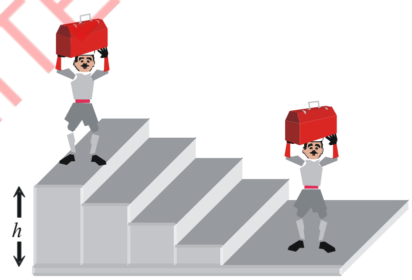


(A) $mg(h+d)+\frac{1}{2}Kd^2$ (B) $mg(h+d)-\frac{1}{2}Kd^2$

(C) $mg(h-d)-\frac{1}{2}Kd^2$ (D) $mg(h-d)+\frac{1}{2}Kd^2$

WORK DEPENDS ON FRAME OF REFERENCE

With change of frame of reference force does not change while displacement may change. So the work done by a force will be different in different frames.



Examples

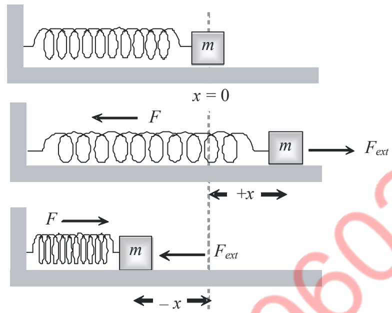
(1) If a porter with a suitcase on his head moves up a staircase, work done by the upward lifting force relative to him will be zero (as displacement relative to him is zero) while relative to a person on the ground will be *mgh*.

(2) If a person is pushing a box inside a moving train, the work done in the frame of train will $\vec{F}.\vec{s}$ while in the frame of earth will be $\vec{F}.(\vec{s}+\vec{s}_0)$ where $\vec{s}_0$ is the displacement of the train relative to the ground. Different types of forces and their corresponding potential energy.

(A) ELASTIC POTENTIAL ENERGY

(1) **Restoring force and spring constant :** When a spring is stretched or compressed from its normal position ($x = 0$) by a small distance x, then a restoring force is produced in the spring to bring it to the normal position.

According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.



i.e. $\vec{F} \propto -\vec{x}$

or $\vec{F} = -k\vec{x}$ ....(i)

where k is called spring constant.

If x = 1, F = k (Numerically)

or $k = F$

Hence spring constant is numerically equal to force required to produce unit displacement (compression or extension) in the spring. If required force is more, then spring is said to be more stiff and vice-versa.

Actually k is a measure of the stiffness/softness of the spring.

Dimension : As $k = \frac{F}{x}$ $\therefore$ $[k] = \frac{[F']}{[x]} = \frac{[MLT^{-2}]}{L} = [MT^{-2}]$

Units : S.I. unit Newton/metre, C.G.S unit Dyne/cm.

Note : ❑ Dimension of force constant is similar to surface tension.

(2) **Expression for elastic potential energy :** When a spring is stretched or compressed from its normal position ($x = 0$), work has to be done by external force against restoring force.

$\vec{F}_{ext} = \vec{F}_{restoring} = k\vec{x}$

Let the spring is further stretched through the distance dx, then work done

$$dW = \vec{F}_{ext}.dx = F_{ext}.dx\cos 0° = kx\,dx \;[\text{As } \cos 0° = 1]$$

Therefore total work done to stretch the spring through a distance x from its mean position is given by

$$W = \int_0^x dW = \int_0^x kx\,dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy of the stretched spring.

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2$

$$U = \frac{1}{2}Fx \qquad \left[\text{As } k = \frac{F}{x}\right]$$

$$U = \frac{F^2}{2k} \qquad \left[x = \frac{F}{k}\right]$$

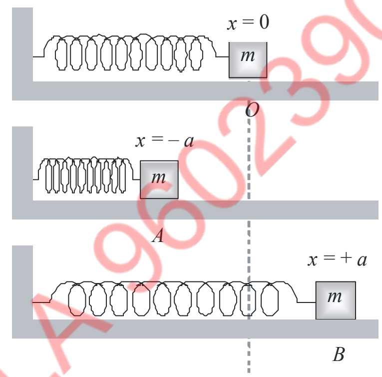
$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

Note : ❑ If spring is stretched from initial position x_1 to final position x_2 then work done

$= $ Increment in elastic potential energy $= \frac{1}{2}k(x_2^2 - x_1^2)$

(3) **Energy graph for a spring :** If the mass attached with spring performs simple harmonic motion about its mean position then its potential energy at any position (*x*) can be given by

$$U = \frac{1}{2}kx^2 \quad \text{...(i)}$$



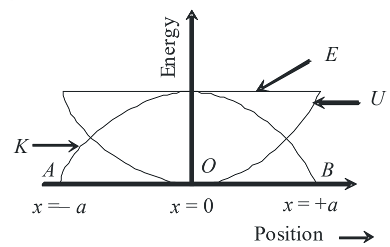
So for the extreme position

$U = \frac{1}{2}ka^2$ [As x = ± a for extreme]

This is maximum potential energy or the total energy of mass.

$\therefore$ Total energy $E = \frac{1}{2}ka^2$ ...(ii)

[Because velocity of mass = 0 at extreme $\therefore$ $K = \frac{1}{2}mv^2 = 0$]



Now kinetic energy at any position $K = E - U = \frac{1}{2}ka^2 - \frac{1}{2}kx^2$

$K = \frac{1}{2}k(a^2 - x^2)$ ...(iii)

From the above formula we can check that

$U_{max} = \frac{1}{2}ka^2$ [At extreme x = ± a] and $U_{min} = 0$ [At mean x = 0]

$K_{max} = \frac{1}{2}ka^2$ [At mean x = 0] and $K_{min} = 0$ [At extreme x = ± a]

$E = \frac{1}{2}ka^2$ = constant (at all positions)

It mean kinetic energy changes parabolically w.r.t. position but total energy remain always constant irrespective to position of the mass

(B) ELECTROSTATIC POTENTIAL ENERGY

It is the energy associated with state of separation between charged particles that interact via electric force. For two point charge q_1 and q_2, separated by distance r.

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

While for a point charge q at a point in an electric field where the potential is V

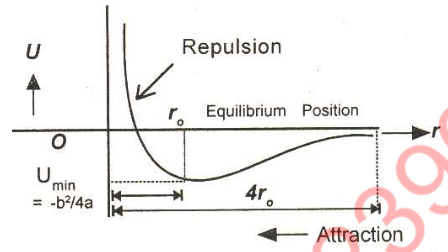
$U = qV$

As charge can be positive or negative, electric potential energy can be positive or negative.

Intermolecular potential energy :

Reference point = ∞ i.e. $P.E_{\infty} = 0$

$$U(r) = \frac{a}{r^{12}} - \frac{b}{r^6}, \qquad F = -\frac{dU}{dr} = \frac{6b}{r^7}\left(\frac{2a/b}{r^6} - 1\right)$$

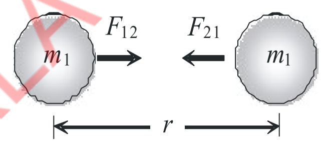


(C) GRAVITATIONAL POTENTIAL ENERGY

It is the usual form of potential energy and is the energy associated with the state of separation between two bodies that interact via gravitational force.

For two particles of masses m_1 and m_2 separated by a distance *r*

Gravitational potential energy $U = -\frac{Gm_1m_2}{r}$



(1) If a body of mass *m* at height *h* relative to surface of earth then

Gravitational potential energy $U = \frac{mgh}{1 + \frac{h}{R}}$

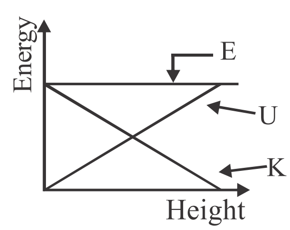
Where R = radius of earth, g = acceleration due to gravity at the surface of the earth.

(2) If h << R then above formula reduces to U = mgh.

(3) If *V* is the gravitational potential at a point, the potential energy of a particle of mass m at that point will be

$U = mV$

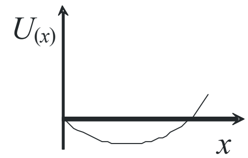
(4) Energy height graph : When a body projected vertically upward from the ground level with some initial velocity then it possesses kinetic energy but its potential energy is zero.

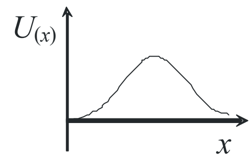


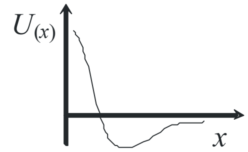
As the body moves upward its potential energy increases due to increase in height but kinetic energy decreases (due to decrease in velocity). At maximum height its kinetic energy becomes zero and potential energy maximum but through out the complete motion total energy remains constant as shown in the figure.

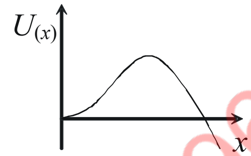
SOLVED EXAMPLE

Example 1. A particle which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F = (-Kx + ax^3)$. Here k and a are positive constants. For , the functional from of the potential energy of the particle is

(A)


(B)


(C)


(D)


Solution : **(D)**

$$F = -\frac{dU}{dx} \Rightarrow dU = -F.dx \quad \Rightarrow U = -\int_0^x (-kx + ax^3)dx \Rightarrow U = \frac{kx^2}{2} - \frac{ax^4}{4}$$

$\therefore$ We get at $x = 0$ and $x = \sqrt{\frac{2k}{a}}$

Also we get negative for $x > \sqrt{\frac{2k}{a}}$

From the given function we can see that F = 0 at x = 0 i.e. slope of U-x graph is zero at x = 0.

Example 2. The potential energy of a body is given by $V = A - Bx^2$ (where x is the displacement). The magnitude of force acting on the particle is

(A) Constant
(B) Proportional to x
(C) Proportional to
(D) Inversely proportional to x

Solution : **(B)**

$$F = \frac{-dU}{dx} = -\frac{d}{dx}(A - Bx^2) = 2B\chi \qquad \therefore F \propto x$$

Example 3. A long spring is stretched by 2 cm, its potential energy is U. If the spring is stretched by 10 cm, the potential energy stored in it will be

(A) U / 25
(B) U / 5
(C) 5 U
(D) 25 U

Solution : **(D)**

Elastic potential energy of a spring $U = \frac{1}{2}kx^2 \;\therefore\; U \propto x^2$

So $\frac{U_2}{U_1} = \left(\frac{x_2}{x_1}\right)^2 \Rightarrow \frac{U_2}{U} = \left(\frac{10cm}{2cm}\right)^2 \Rightarrow U_2 = 25U$

Example 4. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

(A) 6.25 N-m (B) 12.50 N-m

(C) 18.75 N-m (D) 25.00 N-m

Solution : **(C)**

Work done to stretch the spring from x_1 to x_2

$$W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}5\times10^3[(10\times10^{-2}) - (5\times10^{-2})^2] = \frac{1}{2}\times5\times10^3\times75\times10^{-4} = 18.75 \text{ Nm}$$

Example 5. A proton has a positive charge. If two protons are brought near to one another, the potential energy of the system will

(A) Increase (B) Decrease

(C) Remain the same (D) Equal to the kinetic energy

Solution : **(A)**

As the force is repulsive in nature between two protons. Therefore potential energy of the system increases.

Example 6. Two protons are situated at a distance of 100 *fermi* from each other. The potential energy of this system will be in *eV*

(A) 1.44 (B) 1.44×10^3

(C) 1.44×10^2 (D) 1.44×10^4

Solution : **(D)**

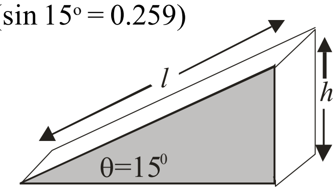
$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{100\times10^{-15}} = 2.304\times10^{-15}\text{ J}$$

$$= \frac{2.304\times10^{-15}}{1.6\times10^{-19}}\text{eV} = 1.44\times10^4\text{ eV}$$

Example 7. The work done in pulling up a block of wood weighing 2kN for a length of 10 m on a smooth plane inclined at an angle of 15^o with the horizontal is ($\sin 15^o = 0.259$)

(A) 4.36 k J (B) 5.17 k J

(C) 8.91 k J (D) 9.82 k J



Solution : **(B)**

Work done = mg × h

$= 2 \times 10^3 \times l \sin\theta$

$= 2 \times 10^3 \times 10 \times \sin 15° = 5176 \text{ J} = 5.17 \text{ kJ}$

Example 8. Two identical cylindrical vessels with their bases at same level each contain a liquid of density d. The height of the liquid in one vessel is h_1 and that in the other vessel is h_2. The area of either vases is A. The work done by gravity in equalizing the levels when the two vessels are connected, is

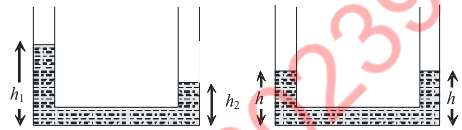
(A) $(h_1 - h_2)gd$ (B) $(h_1 - h_2)\ gAd$

(C) $\frac{1}{2}(h_1 - h_2)^2 gAd$ (D) $\frac{1}{4}(h_1 - h_2)^2 gAd$

Solution : **(D)**

Potential energy of liquid column is given by $mg\frac{h}{2} = Vdg\frac{h}{2} = Ahdg\frac{h}{2} = \frac{1}{2}Adgh^2$

Initial potential energy $= \frac{1}{2}Adgh_1^2 + \frac{1}{2}Adgh_2^2$



Final potential energy $= \frac{1}{2}Adgh^2 + \frac{1}{2}Adh^2g = Adgh^2$

Work done by gravity = change in potential energy

$$W = \left[\frac{1}{2}Adgh_1^2 + \frac{1}{2}Adgh_2^2\right] - Adgh^2$$

$$= Adg\left[\frac{h_1^2}{2} + \frac{h_2^2}{2}\right] - Adg\left(\frac{h_1 + h_2}{2}\right)^2 \quad [\text{As } h = \frac{h_1 + h_2}{2}]$$

$$= Adg\left[\frac{h_1^2}{2} + \frac{h_2^2}{2} - \left(\frac{h_1^2 + h_2^2 + 2h_1h_2}{4}\right)\right] = \frac{Adg}{4}(h_1 - h_2)^2$$

Example 9. If *g* is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an abject of mass *m* raised from the surface of earth to a height equal to the radius of the earth *R*, is

(A) $\frac{1}{2}mgR$ (B) 2mgR

(C) mgR (D) $\frac{1}{4}mgR$

Solution : **(A)**

Work done = gain in potential energy $= \frac{mgh}{1 + h/R} = \frac{mgR}{1 + R/R} = \frac{1}{2}mgR$ [As $h = R$ (given)]

Example 10. The work done in raising a mass of 15 gm from the ground to a table of 1*m* height is

(A) 15 J (B) 152 J

(C) 1500 J (D) 0.15 J

Solution : **(D)**

$W = mgh = 15 \times 10^{-3} \times 10 \times 1 = 0.15\ J.$

Example 11. A body is falling under gravity. When it loses a gravitational potential energy by U, its speed is v. The mass of the body shall be

(A) $\frac{2U}{v}$ (B) $\frac{U}{2v}$

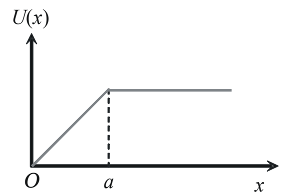
(C) $\frac{2U}{v^2}$ (D) $\frac{U}{2v^2}$

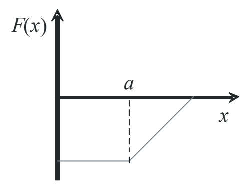
Solution : **(C)**

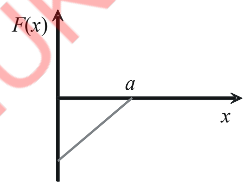
Loss in potential energy = gain in kinetic energy $\Rightarrow U = \frac{1}{2}mv^2 \;\therefore\; m = \frac{2U}{v^2}$.

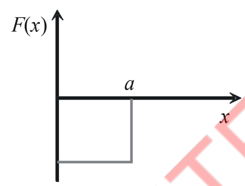
EXERCISE

1. The potential energy of a system is represented in the first figure. The force acting on the system will be represented by



(A)


(B)


(C)


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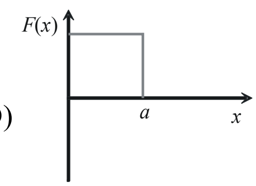


2. A particle moves in a potential region given by $U = 8x^2 - 4x + 400$ J. Its state of equilibrium will be

(A) x = 25 m (B) x = 0.25 m

(C) x = 0.025 m (D) x = 2.5 m

3. Two springs of spring constants 1500 N/m and 3000 N/m respectively are stretched with the same force. They will have potential energy in the ratio

(A) 4 : 1 (B) 1 : 4

(C) 2 : 1 (D) 1 : 2

4. A body is attached to the lower end of a vertical spiral spring and it is gradually lowered to its equilibrium position. This stretches the spring by a length *x*. If the same body attached to the same spring is allowed to fall suddenly, what would be the maximum stretching in this case

(A) x (B) 2x

(C) 3x (D) x/2

5. Two equal masses are attached to the two ends of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass is

(A) $\frac{1}{2}kx^2$ (B) $-\frac{1}{2}kx^2$

(C) $\frac{1}{4}kx^2$ (D) $-\frac{1}{4}kx^2$

6. ${}_{80}Hg^{208}$ nucleus is bombarded by α-particles with velocity 10^7 m/s. If the -particle is approaching the Hg nucleus head-on then the distance of closest approach will be

(A) 1.115×10^{-13} m (B) 11.15×10^{-13} m

(C) 111.5×10^{-13} m (D) Zero

7. A charged particle A moves directly towards another charged particle B. For the system, the total momentum is P and the total energy is E

(A) P and E are conserved if both A and B are free to move

(B) (A) is true only if A and B have similar charges

(C) If B is fixed, E is conserved but not P

(D) If B is fixed, neither E nor P is conserved

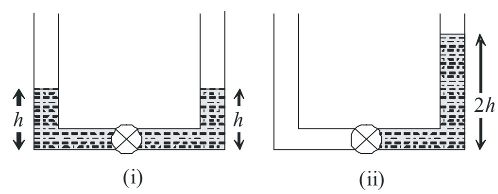
8. A liquid of density d is pumped by a pump P from situation (i) to situation (ii) as shown in the diagram. If the cross-section of each of the vessels is a, then the work done in pumping (neglecting friction effects) is

(A) 2dgh

(B) dgha

(C) $2dgh^2a$

(D) dgh^2a



9. The mass of a bucket containing water is 10 kg. What is the work done in pulling up the bucket from a well of depth 10 m if water is pouring out at a uniform rate from a hole in it and there is loss of 2kg of water from it while it reaches the top $(g = 10\ m/\sec^2)$

(A) 1000 J (B) 800 J

(C) 900 J (D) 500 J

10. A rod of mass m and length l is lying on a horizontal table. The work done in making it stand on one end will be

(A) mgl (B) $\frac{mgl}{2}$

(C) $\frac{mgl}{4}$ (D) 2mgl

There are various form of energy

(i) mechanical energy (ii) chemical energy (iii) electrical energy (iv) magnetic energy

(v) nuclear energy (vi) sound energy (vii) light energy etc

Energy of system always remain constant it can neither be created nor it can be destroyed however it may be converted from one form to another

Example

Law of Conservation of Energy.

(1) Law of conservation of energy

For a body or an isolated system by work-energy theorem we have $K_2 - K_1 = \int \vec{F}.d\vec{r}$ ...(i)

But according to definition of potential energy in a conservative field $U_2 - U_1 = -\int \vec{F}.d\vec{r}$ ...(ii)

So from equation (i) and (ii) we have

$K_2 - K_1 = -(U_2 - U_1)$

or $K_2 + U_2 = K_1 + U_1$

i.e. $K + U = \text{constant}$.

For an isolated system or body in presence of conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depend upon time. This is known as the law of conservation of mechanical energy.

$\Delta(K + U) = \Delta E = 0$ [As + is constant in a conservative field]

$\therefore$ $\Delta K + \Delta U = 0$

i.e. if the kinetic energy of the body increases its potential energy will decrease by an equal amount and vice-versa.

(2) Law of conservation of total energy : If some non-conservative force like friction is also acting on the particle, the mechanical energy is no more constant. It changes by the amount of work done by the frictional force.

$\Delta(K + U) = \Delta E = W_f$ [where W_f is the work done against friction]

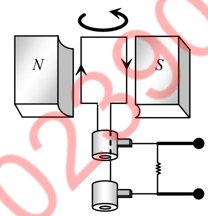
The lost energy is transformed into heat and the heat energy developed is exactly equal to loss in mechanical energy.

We can, therefore, write $\Delta E + Q = 0$ [where Q is the heat produced]

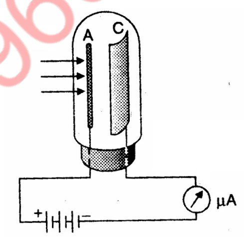
This shows that if the forces are conservative and non-conservative both, it is not the mechanical energy alone which is conserved, but it is the total energy, may be heat, light, sound or mechanical *etc*., which is conserved.

In other words : "*Energy may be transformed from one kind to another but it cannot be created or destroyed. The total energy in an isolated system is constant*". This is the law of conservation of energy.

Electric energy $\xrightarrow{\text{Motor}}$ Mechanical energy



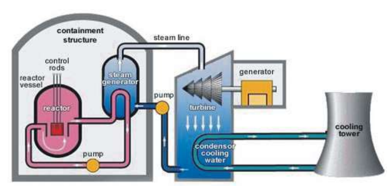
Mechanical energy $\xrightarrow{\text{Generator}}$ Electrical energy



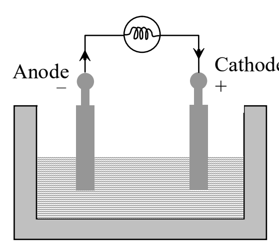
Light energy $\xrightarrow{\text{Photocell}}$ Electrical energy

Electrical energy $\xrightarrow{\text{Heater}}$ Heat energy

Electrical energy $\xrightarrow{\text{Radio}}$ Sound energy

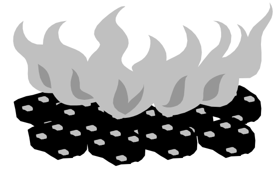


Nuclear energy $\xrightarrow{\text{Nuclear Reactor}}$ Electrical energy

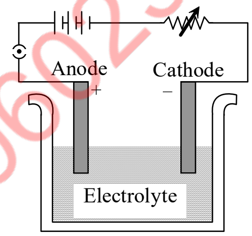


Chemical energy $\xrightarrow{\text{Cell}}$ Electrical energy

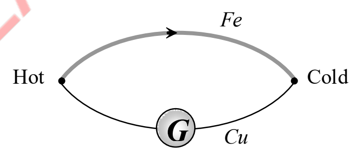
Chemical energy $\xrightarrow{\text{Coal Burning}}$ Heat energy



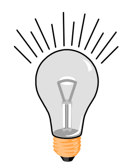
Sound energy $\xrightarrow{\text{Microphone}}$ Electrical energy



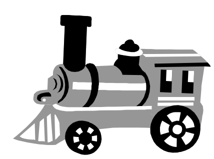
Electrical energy $\xrightarrow{\text{Secondary Cell}}$ Chemical energy



Heat energy $\xrightarrow{\text{Thermo-couple}}$ Light



Electrical energy $\xrightarrow{\text{Bulb}}$ Light energy



Heat energy $\xrightarrow{\text{Engine}}$ Mechanical energy

SOLVED EXAMPLE

Example 1. Two stones each of mass 5*kg* fall on a wheel from a height of 10*m*. The wheel stirs 2*kg* water. The rise in temperature of water would be

(A) 2.6° C (B) 1.2° C

(C) 0.32° C (D) 0.12° C

Solution : **(D)**

For the given condition potential energy of the two masses will convert into heat and temperature of water will increase W = JQ $\Rightarrow$ $2m \times g \times h = J(m_w\ S\ \Delta t) \Rightarrow$

$$2 \times 5 \times 10 \times 10 = 4.2(2 \times 10^3 \times \Delta t)$$

$$\therefore\ \Delta t = \frac{1000}{8.4 \times 10^3} = 0.119°C = 0.12°C$$

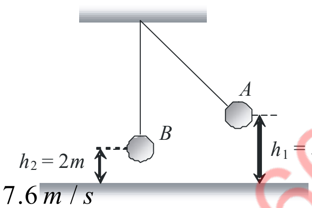
Example 2. A boy is sitting on a swing at a maximum height of 5*m* above the ground. When the swing passes through the mean position which is 2*m* above the ground its velocity is approximately

(A) 7.6 m/s (B) 9.8 m/s

(C) 6.26 m/s (D) None of these

Solution : **(A)**

By the conservation of energy Total energy at point A = Total energy at point B



$\Rightarrow \quad mgh_1 = mgh_2 + \frac{1}{2}mv^2$

$\Rightarrow \quad 9.8 \times 5 = 9.8 \times 2 + \frac{1}{2}v^2$

$\Rightarrow$ v2 = 58.8 ∴ v = 7.6 m/s $v^2 = 58.8 \quad \therefore \quad v = 7.6\,m/s$

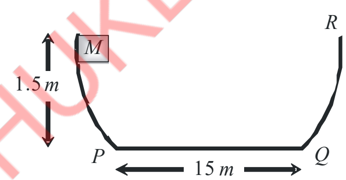
Example 3. A block of mass *M* slides along the sides of a bowl as shown in the figure. The walls of the bowl are frictionless and the base has coefficient of friction 0.2. If the block is released from the top of the side, which is 1.5 m high, where will the block come to rest ? Given that the length of the base is 15 m

(A) 1 *m* from *P*

(B) Mid point

(C) 2 *m* from *P*

(D) At *Q*



Solution : **(B)**

Potential energy of block at starting point = Kinetic energy at point P = Work done against friction in traveling a distance s from point P.

$\therefore \quad mgh = \mu\, mgs \quad \Rightarrow s = \frac{h}{\mu} = \frac{1.5}{0.2} = 7.5\text{m}$

i.e. block come to rest at the mid point between P and Q.

Example 4. If we throw a body upwards with velocity of 4 ms^{-1} at what height its kinetic energy reduces to half of the initial value? Take g = 10 m/s^2

(A) 4m (B) 2 m

(C) 1 m (D) None of these

Solution : **(D)**

We know kinetic energy $K = \frac{1}{2}mv^2 \quad \therefore \quad v \propto \sqrt{K}$

When kinetic energy of the body reduces to half its velocity becomes

$v = \frac{u}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\ m/s$

From the equation $v^2 = u^2 - 2gh \Rightarrow (2\sqrt{2})^2 = (4)^2 - 2 \times 10h \qquad \therefore \ h = \frac{16-8}{20} = 0.4\text{m}$.

Example 5. A 2kg block is dropped from a height of 0.4 m on a spring of force constant $K = 1960\ Nm^{-1}$. The maximum compression of the spring is

(A) 0.1 m (B) 0.2 m

(C) 0.3 m (D) 0.4 m

Solution : **(A)**

When a block is dropped from a height, its potential energy gets converted into kinetic energy and finally spring gets compressed due to this energy.

$\therefore$ Gravitational potential energy of block = Elastic potential energy of spring

$$\Rightarrow \quad mgh = \frac{1}{2}Kx^2 \Rightarrow x = \sqrt{\frac{2mgh}{K}} = \sqrt{\frac{2\times2\times10\times0.4}{1960}} = 0.09\ m \approx 0.1\ m$$

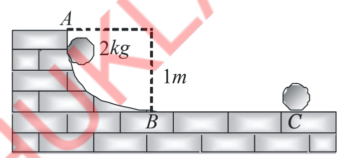
Example 6. A block of mass 2kg is released from A on the track that is one quadrant of a circle of radius 1m. It slides down the track and reaches B with a speed of and finally stops at C at a distance of 3m from B. The work done against the force of friction is

(A) 10 J

(B) 20 J

(C) 2 J

(D) 6 J



Solution : **(B)**

Block possess potential energy at point A = mgh

Finally block stops at point C. So its total energy goes against friction i.e. work done against friction is 20 J.

Example 7. A stone projected vertically upwards from the ground reaches a maximum height h. When it is at a height the ratio of its kinetic and potential energies is

(A) 3 : 4 (B) 1 : 3

(C) 4 : 3 (D) 3 : 1

Solution : **(B)**

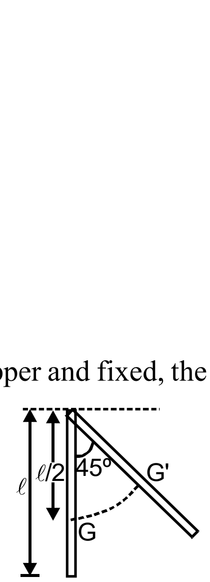
At the maximum height, Total energy = Potential energy = mgh

At the height , Potential energy =

and Kinetic energy = Total energy – Potential energy

EXAMPLE

Example 1. A meter scale of mass m initially vertical is displaced at 45º keeping the upper and fixed, the charge in PE will be-



Solution. Work = change in PE = Force × displacement

$$dU = mg\frac{\ell}{2}(1-\cos\theta)$$

$= mg \times \frac{1}{2}(1 - \cos 45^\circ) \qquad (\because \ell = 1m)$

$= \frac{mg}{2}\left(1 - \frac{1}{\sqrt{2}}\right)$

Example 2. If the speed of a car increases 4 times, the stopping distance for this will increase by -

Solution. Work = Change in KE

$\therefore \quad FS = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$

$\frac{S'}{S} = \frac{v'^2}{v^2}$

$\Rightarrow \quad \frac{S'}{S} = 16$

$\Rightarrow \quad S' = 16\,S$

Example 3. If the potential energy function for a particle is $U = a - \frac{b}{x} + \frac{c}{x^2}$ the force constant for oscillation will be.

Solution. $U = a - \frac{b}{x} + \frac{c}{x^2}$ ....(1)

$\therefore \quad \frac{dU}{dx} = -\frac{b}{x^2} - \frac{2c}{x^3}$ ....(2)

and $\frac{d^2U}{dx^2} = \frac{1}{x^3}\left(-2b + \frac{6c}{x}\right)$ ....(3)

` for equilibrium

$\frac{dU}{dx} = 0 \quad \therefore \quad x = \frac{2c}{b}$

$\therefore$ Substituting this value in (3)

$\frac{d^2U}{dx^2} = \left(\frac{b}{2c}\right)^3 \left[-2b + \frac{6c}{2c/b}\right] = \frac{b^4}{8c^3}$

as $\frac{d^2U}{dx^2} = K \quad \therefore \quad K = b^4/8c^3$

Example 4. On passing through a wooden sheet a bullet looses 1/20 of initial velocity. The minimum number of sheets required to completely stop the bullet will be-

Solution. Use $v^2 = u^2 + 2as$

for a sheet of thickness s $v = (19/20)u$

$$\left(\frac{19}{20}u\right) = u^2 + 2as$$

$$2as = (361/400)u^2 - u^2 \quad a = -\left(\frac{39}{400}\right)\frac{u^2}{2s}$$

suppose for n sheet v = 0

$$\therefore \quad 0^2 = u^2 + 2a\,(ns) \quad n = -\frac{u^2}{2as} = \frac{u^2}{2\left(\frac{39}{400}\right)\frac{u^2}{2s}\times s} \approx 11$$

Example 5. The work done in taking out 2 lit of water using a bucket of mass 0.5 kg from a well of depth 6m will be-

Solution. $W = mgh$

$= (m_{bucket} + m_{water})gh$ [2 Lit water = 2 kg water]

$= (0.5 + 2.00) \times 9.8 \times 6$

$= 15 \times 9.8 = 147$ J

Example 6. A body has velocity 200 m/s and its kinetic energy is 200 J. The mass of the body would be

Solution. $\frac{1}{2}mv^2 = E$

or $m = \left(\frac{2E}{v^2}\right) = \frac{2\times 200}{(200)^2}$

$= \frac{4\times 10^2}{4\times 10^4} = \frac{1}{100}$

$\therefore$ m = 0.01 kg

Example 7. A body falls on the surface of the earth from a height of 20 cm. If after colliding with the earth, its mechanical energy is lost by 75%, then body would reach upto a height of

Solution. $\frac{1}{4}mgh = mgh'$

$\therefore \quad h' = \frac{h}{4} = \frac{1}{4}\times 20 = 5\text{cm}$

Example 8. Potential energy function describing the interaction between two atoms of a diatomic molecule is $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

In stable equilibrium, the distance between them would be

Solution. In stable equilibrium potential energy is minimum. For minimum value of U(x)

$$\frac{d}{dx}[U(x)] = 0$$

or $\frac{d}{dx}\left(\frac{a}{x^{12}} - \frac{b}{x^6}\right) = 0$ or $\frac{-12a}{x^{13}} + \frac{6b}{x^7} = 0$

or $\frac{6}{x^{13}}(-2a + bx^6) = 0$ or $bx^6 - 2a = 0$

$\therefore$ $x = \left(\frac{2a}{b}\right)^{1/6}$

Example 9. Two electrons are at a distance of 1×10^{-12}m from each other. Potential energy (in eV) of this system would be

Solution. Potential energy of the system

$$U = \frac{Kq_1q_2}{r}$$

$$= \frac{9\times10^9 \times 1.6\times10^{-19} \times 1.6\times10^{-19}}{1\times10^{-12}} = 23.04 \times 10^{-17} \text{ Joule}$$

$$= \frac{23.04\times10^{-17}}{1.6\times10^{-19}}\text{eV} = 1.44\times10^3 \text{ eV}$$

Example 10. Potential energy function U(r) corresponding to the central force $F = \frac{K}{r^2}$ would be

Solution. Central force is conservative. Therefore

$$\vec{F}(r) = -\nabla U = -\frac{dU}{dr}\hat{r}$$

or $dU = -\vec{F}(r).\vec{d}r = -F(r)dr$

$\therefore$ $U = \int dU = -\int F(r)\,dr = -\int \frac{k}{r^2}dr$ $\Rightarrow \frac{k}{r} + C$

$$= -K\int \frac{1}{r^2}dr = Kr^{-1} + C$$

If at $r = \infty$, U = 0, then C = 0

$$U = Kr^{-1} = \frac{K}{r}$$

Example 11. The stopping distance for a vehicle of mass M moving with speed v along level road, will be (μ is the coefficient of friction between tyres and the road)

Solution. When the vetical of mass m is moving with velocity v, the kinetic energy of the where $K = 1/2\ mv^2$ and if S is the stopping distance, work done by the friction

$W = FS \cos\theta = m\ MgS \cos 180^o = - m\ MgS$

So by Work-Energy theorem, $W = \Delta K = K_f - k_i$

$\Rightarrow \quad -\mu\ MgS = 0 - 1/2\ Mv^2$

$$\Rightarrow \quad S = \frac{v^2}{2\mu g}$$

Example 12. A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force equal to $(-k/r^2)$, where k is constant. The total energy of the particle is

Solution. As the particle is moving in a circle, so

$$\frac{mv^2}{r} = \frac{k}{r^2} \qquad \text{Now} \qquad K.E = \frac{1}{2}mv^2 = \frac{k}{2r}$$

Now as $F = -\dfrac{dU}{dr}$ P.E, $U = -\displaystyle\int_{\infty}^{r} F\,dr$

$$= \int_{\infty}^{r} +\left(\frac{k}{r^2}\right)dr = -\frac{k}{r}$$

So total energy = P.E + K.E.

$$= -\frac{k}{r} + \frac{k}{2r} = -\frac{k}{2r}$$

Negative energy means that particle is in bound state.

EXERCISE

1. A particle of mass 'm' and charge 'q' is accelerated through a potential difference of 'V' volt. Its energy is

(A) qV (B) mqV

(C) $\left(\dfrac{q}{m}\right)V$ (D) $\dfrac{q}{mV}$

2. An ice cream has a marked value of 700 kcal. How many kilowatt hour of energy will it deliver to the body as it is digested

(A) 0.81 kWh (B) 0.90 kWh

(C) 1.11 kWh (D) 0.71 kWh

3. A metallic wire of length *L metres* extends by *l metres* when stretched by suspending a weight *Mg* to it. The mechanical energy stored in the wire is

(A) 2 Mgl (B) Mgl

(C) $\dfrac{Mgl}{2}$ (D) $\dfrac{Mgl}{4}$

4. The work done by a person in carrying a box of mass 10 kg. through a vertical height of 10 m is 4900J. The mass of the person is

(A) 49 kg (B) 10 kg

(C) 40 kg (D) 98 kg

5. A uniform rod of length 4m and mass 20kg is lying horizontal on the ground. The work done in keeping it vertical with one of its ends touching the ground, will be -

(A) 20 J (B) 392 J

(C) 98 J (D) 390 J .

6. A man throws the bricks to the height of 12 m where they reach with a speed of 12 m/sec. If he throws the bricks such that they just reach this height, what percentage of energy will he save

(A) 38% (B) 40%

(C) 20% (D) 72%

7 A body of mass 8 kg moves under the influence of a force. The position of the body and time are related as $x = 1/2t^2$ where x is in meter and t in sec. The work done by the force in first two seconds.

(A) 20 J (B) 16 J

(C) 8 J (D) 32 J .

POWER

Power of a body is defined as the rate at which the body can do the work.

Average power $\bar{P} = \frac{\Delta w}{\Delta t}$, Where Δw is the work done in time Δt

Instantaneous power $P_{inst} = \frac{dw}{dt}$; Its value may change with time.

i.e. power is equal to the scalar product of force with velocity. $\left(P = \vec{F}.\vec{V}\right)$

Important points

(1) Dimension : $[M^1L^2T^{-3}]$

(2) Units : Watt or Joule/sec [S.I.]

Erg/sec [C.G.S.]

Practical units : Kilowatt (kW), Mega watt (MW) and Horse power (hp)

Relations between different units :

1 watt =1 joule/sec $= 10^7$ erg / sec

1 kW = 10^3 W

1 Meg watt = 10^6 W

1 HP = 746 W

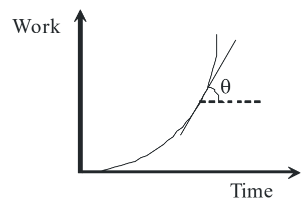
(3) If work done by the two bodies is same then power $P \alpha \frac{1}{\text{time}}$

i.e. the body which performs the given work in lesser time possesses more power and vice-versa.

(4) As power = work/time, any unit of power multiplied by a unit of time gives unit of work (or energy) and not power, *i.e.* Kilowatt-hour or watt-day are units of work or energy.

$$1\text{kWh} = 10^3 \times \frac{\text{J}}{\text{sec}} \times (60 \times 60\,\text{sec}) = 3.6 \times 10^6\,\text{J}$$

(5) The slope of work time curve gives the instantaneous power. As P = dW/dt = tanθ



(6) Area under power time curve gives the work done as $P = \frac{dw}{dt}$

$\therefore \quad W = \int P\,dt$

$\therefore \quad$ W = Area under P-t curve

SOLVED EXAMPLE

Example 1. A car of mass '*m*' is driven with acceleration '*a*' along a straight level road against a constant external resistive force '*R*'. When the velocity of the car is '*v*', the rate at which the engine of the car is doing work will be

(A) Rv (B) mav

(C) $(R + ma)v$ (D) $(ma - R)v$

Solution : **(C)**

The engine has to do work against resistive force *R* as well as car is moving with acceleration *a*.

Power = Force × velocity = (*R*+*ma*)*v*.

Example 2. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed *v*, the electrical power output will be proportional to

(A) v (B) v^2

(C) v^3 (D) v^4

Solution : **(C)**

Force $v\frac{dm}{dt} = v\frac{d}{dt}(V \times \rho) = v\rho\frac{d}{dt}[A \times l] = v\rho A\frac{dl}{dt} = \rho A v^2$

Power = F × v = $\rho A v^2 \times v = \rho A v^3$ $\therefore$ $P \propto v^3$

Example 3. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain twice as much water from the same pipe in the same time, power of the motor has to be increased to

(A) 16 times (B) 4 times

(C) 8 times (D) 2 times

Solution : **(D)**

$$P = \frac{\text{work done}}{\text{time}} = \frac{mgh}{t} \quad \therefore \; P \propto m$$

i.e. To obtain twice water from the same pipe in the same time, the power of motor has to be increased to 2 times.

Example 4. A force applied by an engine of a train of mass 2.05×10^6 kg changes its velocity from 5 *m/s* to 25 m/s in 5 minutes. The power of the engine is

(A) 1.025 MW (B) 2.05 MW

(C) 5MW (D) 5 MW

Solution : **(B)**

$$\text{Power} = \frac{\text{work done}}{\text{time}} = \frac{\text{increase in kinetic energy}}{\text{time}} = \frac{\frac{1}{2}m\left(v_2^2 - v_1^2\right)}{t} = \frac{\frac{1}{2}\times 2.05\times 10^6 \times [25^2 - 5^2]}{5\times 60}$$

$= 2.05 \times 10^6$ watt $= 2.05$ MW

Example 5. From a water fall, water is falling at the rate of 100 *kg/s* on the blades of turbine. If the height of the fall is 100*m* then the power delivered to the turbine is approximately equal to

(A) 100kW (B) 10 kW

(C) 1kW (D) 1000 kW

Solution : **(A)**

$$\text{Power} = \frac{\text{work done}}{t} = \frac{mgh}{t} = 100\times 10\times 100 = 10^5 \text{ watt} = 100\text{kW} \quad \left[\text{As } \frac{m}{t} = 100\,\frac{kg}{\text{sec}} \text{(given)}\right]$$

Example 6. A particle moves with a velocity $\vec{v} = 5\hat{i} - 3\hat{j} + 6\hat{k}\ ms^{-1}$ under the influence of a constant force $\vec{F} = 10\hat{i} + 10\hat{j} + 20\hat{k}$ N. The instantaneous power applied to the particle is

(A) 200 J-s^{-1} (B) 40 J-s^{-1}

(C) 140 J-s^{-1} (D) 170 J-s^{-1}

Solution : **(C)**

$$P = \vec{F}.\vec{v} = (10\hat{i} + 10\hat{j} + 20\hat{k}).(5\hat{i} - 3\hat{j} + 6\hat{k}) = 50 - 30 + 120 = 140 \text{ J} - s^{-1}$$

EXERCISE

1. A car of mass 1250 kg experience a resistance of 750 N when it moves at $30ms^{-1}$. If the engine can develop 30kW at this speed, the maximum acceleration that the engine can produce is

(A) $0.8\ ms^{-2}$ (B) $0.2\ ms^{-2}$

(C) $0.4\ ms^{-1}$ (D) $0.5\ ms^{-2}$

2. A bus weighing 100 quintals moves on a rough road with a constant speed of $72 km/h$. The friction of the road is 9% of its weight and that of air is 1% of its weight. What is the power of the engine. Take $g = 10m/s^2$

(A) 50 kW (B) 100 kW

(C) 150 kW (D) 200 kW

3. Two men with weights in the ratio 5 : 3 run up a staircase in times in the ratio 11 : 9. The ratio of power of first to that of second is

(A) $\frac{15}{11}$ (B) $\frac{11}{15}$

(C) $\frac{11}{9}$ (D) $\frac{9}{11}$

4. A dam is situated at a height of 550 *metre* above sea level and supplies water to a power house which is at a height of 50 *metre* above sea level. 2000 *kg* of water passes through the turbines per second. The maximum electrical power output of the power house if the whole system were 80% efficient is

(A) 8 MW (B) 10 MW

(C) 12.5 MW (D) 16 MW

5. A constant force F is applied on a body. The power (P) generated is related to the time elapsed (t) as

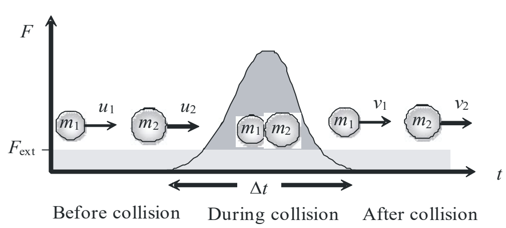
(A) $P \propto t^2$ (B) $P \propto t$

(C) $P \propto \sqrt{t}$ (D) $P \propto t^{3/2}$

COLLISION

Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

In collision particles may or may not come in real touch e.g. in collision between two billiard balls or a ball and bat there is physical contact while in collision of alpha particle by a nucleus (*i.e.* Rutherford scattering experiment) there is no physical contact.



(1) **Stages of collision :** There are three distinct identifiable stages in collision, namely, before, during and after. In the before and after stage the interaction forces are zero. Between these two stages, the interaction forces are very large and often the dominating forces governing the motion of bodies. The magnitude of the interacting force is often unknown, therefore, Newton's second law cannot be used, and the law of conservation of momentum is useful in relating the initial and final velocities.

(2) **Momentum and energy conservation in collision :**

(i) Momentum conservation : In a collision the effect of external forces such as gravity or friction are not taken into account as due to small duration of collision (Δt) average impulsive force responsible for collision is much larger than external force acting on the system and since this impulsive force is 'Internal' therefore the total momentum of system always remains conserved.

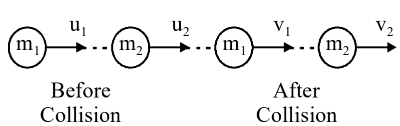
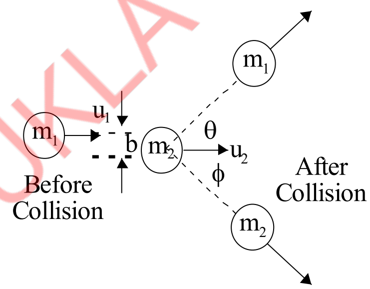
(ii) Energy conservation : In a collision 'total energy' is also always conserved. Here total energy includes all forms of energy such as mechanical energy, internal energy, excitation energy, radiant energy or even mass energy.

These laws are the fundamental laws of physics and applicable for any type of collision but this is not true for conservation of kinetic energy.

(3) **Types of collision :** (i) On the basis of conservation of kinetic energy.

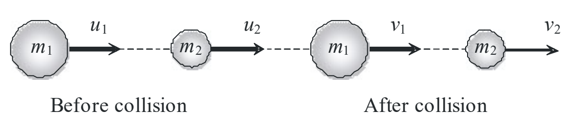
Perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic.	If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to inelastic.	If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.
Coefficient of restitution $e = 1$	Coefficient of restitution $0 < e < 1$	Coefficient of restitution $e = 0$
$(KE)_{final} = (KE)_{initial}$	Here kinetic energy appears in other forms. In some cases $(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released	The term 'perfectly inelastic' does not necessarily mean that all the initial kinetic energy is lost, it implies that the loss in kinetic energy is as large as it can be. (Consistent with momentum conservation).
Examples : (1) Collision between atomic particles (2) Bouncing of ball with same velocity after the collision with earth.	*Examples* : (1) Collision between two billiard balls. (2) Collision between two automobile on a road. In fact all majority of collision belong to this category.	*Example* : Collision between a bullet and a block of wood into which it is fired. When the bullet remains embeded in the block.

(ii) On the basis of the direction of colliding bodies

Head on or one dimensional collision	Oblique collision
In a collision if the motion of colliding particle such before and after the collision along the same collision line the collision is said to be head on or one dimensional.	If two particle collision is 'glancing' i.e. that their directions of motion after is called oblique. In oblique collision the particles before an after collision are in same plane, the collision: called 2-dimensional otherwise 3-dimensional.
Impact parameter b is zero for this type of collision	Impact parameter b lies between 0 and $r_1 + r_2$ i.e. $0 < b < (r_1 + r_2)$ where r_1 and r_2 a radii of colliding bodies
	
Example : collision of two gliders on an air track	Example : Collision of billiard balls

PERFECTLY ELASTIC HEAD ON COLLISION

Let two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 in the same direction and they collide such that after collision their final velocities are v_1 and v_2 respectively.



According to law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{...(i)}$$

$$\Rightarrow \quad m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{...(ii)}$$

According to law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...(iii)}$$

$$\Rightarrow \quad m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad \text{...(iv)}$$

Dividing equation (iv) by equation (ii)

$$v_1 + u_1 = v_2 + u_2 \qquad ...(v)$$

$$\Rightarrow \qquad u_1 - u_2 = v_2 - v_1 \qquad ...(vi)$$

Relative velocity of approach = Relative velocity of separation

Note :

- The ratio of relative velocity of separation and relative velocity of approach is defined as coefficient of restitution.

  $$e = \frac{v_2 - v_1}{u_1 - u_2} \quad \text{or} \quad v_2 - v_1 = e(u_1 - u_2)$$

- For perfectly elastic collision

  $e = 1 \quad \therefore v_2 - v_1 = u_1 - u_2$ (As shown in eq. (vi)

- For perfectly inelastic collision

  $e = 0 \quad \therefore v_2 - v_1 = 0$ or $v_2 = v_1$ or $v_2 = v_1$

  It means that two body stick together and move with same velocity.

- For inelastic collision

  $0 < e < 1$

  $\therefore$ In short we can say that e is the degree of elasticity of collision and it is dimension less quantity.

Further from equation (v) we get $v_2 = v_1 + u_1 - u_2$

Substituting this value of in equation (i) and rearranging we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \quad ......(vii)$$

Similarly we get $$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2} \quad ......(viii)$$

(1) **Special cases of head on elastic collision**

(i) **If projectile and target are of same mass i.e. $m_1 = m_2$**

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2}{m_1 + m_2}u_2$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

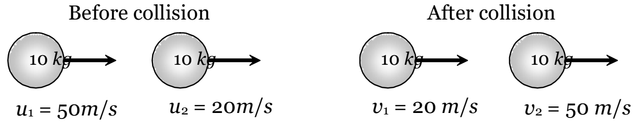
Substituting $m_1 = m_2$ we get

$v_1 = u_2$ and $v_2 = u_1$

It means when two bodies of equal masses undergo head on elastic collision, their velocities get interchanged.

Example : Collision of two billiard balls

Before collision After collision



Sub case : $u_2 = 0$ *i.e.* target is at rest

$v_1 = 0$ $v_2 = u_1$

(ii) **If massive projectile collides with a light target i.e. $m_1 >> m_2$**

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

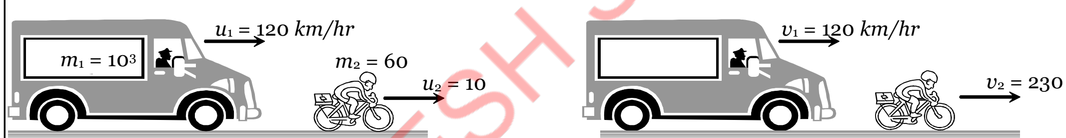
Substituting $m_2 = 0$, we get

$v_1 = u_1$ and $v_2 = 2u_1 - u_2$

Example : Collision of a truck with a cyclist

Sub case : $u_2 = 0$ i.e. target is at rest

$v_1 = u_1$ and $v_2 = 2u_1$



Before collision After collision

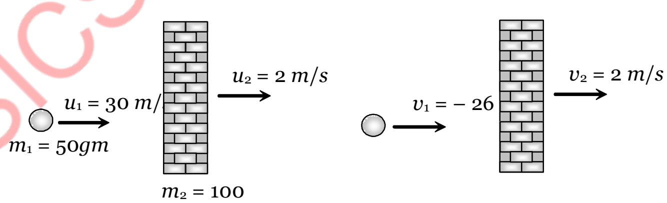
(iii) **If light projectile collides with a very heavy target i.e. $m_1 << m_2$**

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

Substituting $m_1 = 0$, we get

$v_1 = -u_1 + 2u_2$ and $v_2 = u_2$

Example : Collision of a ball with a massive wall.



Before collision After collision

Sub case : $u_2 = 0$ i.e. target is at rest

$v_1 = -u_1$ and $v_2 = 0$

i.e. the ball rebounds with same speed in opposite direction when it collide with stationary and very massive wall.

(2) **Kinetic energy transfer during head on elastic collision**

Kinetic energy of projectile before collision $K_i = \frac{1}{2}m_1u_1^2$

Kinetic energy of projectile after collision $K_f = \frac{1}{2}m_1v_1^2$

Kinetic energy transferred from projectile to target ΔK = decrease in kinetic energy in projectile

$$\Delta K = \frac{1}{2}m_1u_1^2 = -\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1(u_1^2 - v_1^2)$$

Fractional decrease in kinetic energy $\frac{\Delta K}{K} = \frac{\frac{1}{2}m_1(u_1^2 - v_1^2)}{\frac{1}{2}m_1u_1^2} = 1 - \left(\frac{v_1}{u_1}\right)^2$ ...(i)

We can substitute the value of v_1 from the equation $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$

If the target is at rest *i.e.* $u_2 = 0$ then $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$

From equation (i) $\frac{\Delta K}{K} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$ ...(ii)

or $\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1 + m_2)^2}$ ...(iii)

or $\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1 - m_2)^2 + 4m_1m_2}$ ...(iv)

Note :

- Greater the difference in masses less will be transfer of kinetic energy and vice versa
- Transfer of kinetic energy will be maximum when the difference in masses is minimum

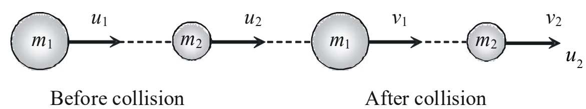
  i.e. $m_1 - m_2 = 0$ or $m_1 = m_2$ then $\frac{\Delta K}{K} = 1 = 100\%$

  So the transfer of kinetic energy in head on elastic collision (when target is at rest) is maximum when the masses of particles are equal i.e. mass ratio is 1 and the transfer of kinetic energy is 100%.

- If $m_2 = nm_1$ then from equation (iii) we get $\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$
- Kinetic energy retained by the projectile $\left(\frac{\Delta K}{K}\right)_{Retained} = 1 -$ kinetic energy transferred by projectile

$$\Rightarrow \left(\frac{\Delta K}{K}\right)_{Retained} = 1 - \left[1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2\right] = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

(3) **Velocity, momentum and kinetic energy of stationary target after head on elastic collision**



(i) Velocity of target : We know $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

$\Rightarrow \quad v_2 = \frac{2m_1u_1}{m_1 + m_2} = \frac{2u_1}{1 + m_2 / m_1}$ [As $u_2 = 0$ and Let $\frac{m_2}{m_1} = n$]

$\therefore \quad v_2 = \frac{2u_1}{1+n}$

(ii) Momentum of target : $P_2 = m_2v_2 = \frac{2nm_1u_1}{1+n}$ $\left[\text{As } m_2 = m_1n \text{ and } v_2 = \frac{2u_1}{1+n}\right]$

$\therefore \quad P_2 = \frac{2m_1u_1}{1+(1/n)}$

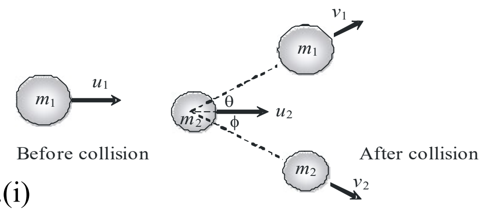
(iii) Kinetic energy of target : $K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}nm_1\left(\frac{2u_1}{1+n}\right)^2 = \frac{2m_1u_1^2n}{(1+n)^2}$

$= \frac{4(K_1)n}{(1-n)^2 + 4n}$ $\left[\text{As } K_1 = \frac{1}{2}m_1u_1^2\right]$

(iv) Relation between masses for maximum velocity, momentum and kinetic energy

Velocity	$v_2 = \frac{2u_1}{1+n}$	For v_2 to be maximum n must be minimum *i.e.* $n = \frac{m_2}{m_1} \to 0 \therefore m_2 << m_1$	Target should be very light.
Momentum	$P_2 = \frac{2m_1u_1}{(1+1/n)}$	For P_2 to be maximum, $(1/n)$ must be minimum or n must be maximum. *i.e.* $n = \frac{m_2}{m_1} \to \infty \therefore m_2 >> m_1$	Target should be massive.
Kinetic energy	$K_2 = \frac{4K_1n}{(1-n)^2 + 4n}$	For K_2 to be maximum $(1-n)^2$ must be minimum. *i.e.* $1 - n = 0 \Rightarrow n = 1 = \frac{m_2}{m_1} \therefore$ $m_2 = m_1$	Target and projectile should be of equal mass.

Perfectly Elastic Oblique Collision



Let two bodies moving as shown in figure.

By law of conservation of momentum

Along *x*-axis, $m_1u_1 + m_2u_2 = m_1v_1 \cos\theta + m_2v_2 \cos\phi$ ...(i)

Along *y*-axis, $0 = m_1v_1 \sin\theta - m_2v_2 \sin\phi$ ...(ii)

By law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...(iii)}$$

In case of oblique collision it becomes difficult to solve problem when some experimental data are provided as in these situations more unknown variables are involved than equations formed.

Special condition : If $m_1 = m_2$ and $u_2 = 0$ substituting these values in equation (i), (ii) and (iii) we get

$u_1 = v_1 \cos\theta + v_2 \cos\phi$ ...(iv)

$0 = v_1 \sin\theta - v_2 \sin\phi$ ...(v)

and $u_1^2 = v_1^2 + v_2^2$ ...(vi)

Squaring (iv) and (v) and adding we get

$$u_1^2 = v_1^2 + v_2^2 + 2v_1u_2\cos(\theta + \phi) \quad \text{...(vii)}$$

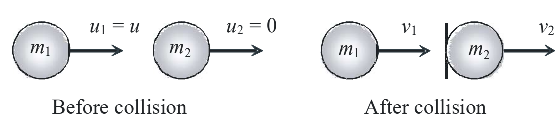
Using (vi) and (vii) we get $\cos(\theta + \phi) = 0$

$\therefore \quad \theta + \phi = \pi/2$

i.e. after perfectly elastic oblique collision of two bodies of equal masses (if the second body is at rest), the scattering angle $\theta + \phi$ would be 90°.

Head on Inelastic Collision.

(1) **Velocity after collision :** Let two bodies *A* and *B* collide inelastically and coefficient of restitution is *e*.



Where $e = \dfrac{v_2 - v_1}{u_1 - u_2} = \dfrac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$

$\Rightarrow \quad v_2 - v_1 = e(u_1 - u_2)$

$\therefore \quad v_2 = v_1 + e(u_1 - u_2)$ …(i)

From the law of conservation of linear momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ …(ii)

By solving (i) and (ii) we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2$$

Similarly $$v_2 = \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2$$

By substituting e = 1, we get the value of v_1 and u_2 for perfectly elastic head on collision.

(2) **Ratio of velocities after inelastic collision :** A sphere of mass *m* moving with velocity *u* hits inelastically with another stationary sphere of same mass.

$$\therefore \quad e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$$

$$\Rightarrow \quad v_2 - v_1 = eu \quad \ldots(i)$$

By conservation of momentum :

Momentum before collision = Momentum after collision

$$mu = mv_1 + mv_2$$

$$\Rightarrow \quad v_1 + v_2 = u \quad \ldots\ldots(ii)$$

Solving equation (i) and (ii) we get $v_1 = \frac{u}{2}(1-e)$ and $v_2 = \frac{u}{2}(1+e)$

$$\therefore \quad \frac{v_1}{v_2} = \frac{1-e}{1+e}$$

(3) **Loss in kinetic energy**

Loss (ΔK) = Total initial kinetic energy – Total final kinetic energy

$$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

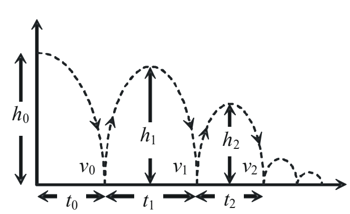
Substituting the value of v_1 and v_2 from the above expression

$$\text{Loss } (\Delta K) = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(1-e^2)(u_1 - u_2)^2$$

By substituting e = 1 we get ΔK = 0 i.e. for perfectly elastic collision loss of kinetic energy will be zero or kinetic energy remains constant before and after the collision.

Rebounding of Ball After Collision With Ground.

If a ball is dropped from a height *h* on a horizontal floor, then it strikes with the floor with a speed.



$v_0 = \sqrt{2gh_0}$ [From $v_2 = u_2 + 2gh$]

and it rebounds from the floor with a speed

$v_1 = ev_0 = e\sqrt{2gh_0}$ $\left[\text{As } e = \dfrac{\text{velocity after collision}}{\text{velocity before collision}}\right]$

(1) **First height of rebound :** $h_1 = \dfrac{v_1^2}{2g} = e^2 h_0$

$\therefore$ $h_1 = e^2 h_0$

(2) **Height of the ball after n^th rebound :** Obviously, the velocity of ball after *n*^th rebound will be

$v_n = e^n v_0$

Therefore the height after n^th rebound will be $h_n = \dfrac{v_n^2}{2g} = e^{2n} h_0$

$\therefore$ $h_n = e^{2n} h_0$

(3) **Total distance travelled by the ball before it stops bouncing**

$H = h_0 + 2h_1 + 2h_2 + 2h_3 + ... = h_0 + 2e^2h_0 + 2e^4h_0 + 2e^6h_0 + ...$

$H = h_0(1 + e^2 + 2^4 + e^4 + e^6 ...)] = h_0\left[1 + 2e^2\left(\dfrac{1}{1-2^2}\right)\right]$ $\left[\text{As } 1 + e^2 + e^4 + ... = \dfrac{1}{1-e^2}\right]$

$\therefore$ $H = h_0\left[\dfrac{1+e^2}{1-e^2}\right]$

(4) **Total time taken by the ball to stop bouncing**

$T = t_0 + 2t_1 + 2t_2 + 2t_3 + ... \sqrt{\dfrac{2h_0}{g}} + 2\sqrt{\dfrac{2h_1}{g}} + 2\sqrt{\dfrac{2h_2}{g}} + ...$

$= \sqrt{\dfrac{2h_0}{g}}[1 + 2e + 2e^2 + ...]$ [As $h_1 = e2h_0$; $h_2 = e^4h_0$]

$= \sqrt{\dfrac{2h_0}{g}}[1 + 2e(1 + e + e^2 + e^3 .....0] = \sqrt{\dfrac{2h_0}{g}}\left[1 + 2e\left(\dfrac{1}{1-e}\right)\right] = \sqrt{\dfrac{2h_0}{g}}\left(\dfrac{1+e}{1-e}\right)$

$\therefore$ $T = \sqrt{\dfrac{2h_0}{g}}\left(\dfrac{1+e}{1-e}\right)$

Perfectly Inelastic Collision.

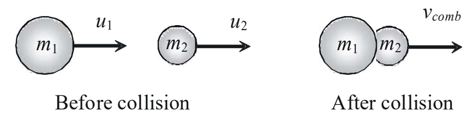
In such types of collisions the bodies move independently before collision but after collision as a one single body.

(1) **When the colliding bodies are moving in the same direction**

By the law of conservation of momentum

$m_1u_1 + m_2u_2 = (m_1 + m_2)v_{comb}$

$\Rightarrow \quad v_{comb} = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2}$



Loss in kinetic energy $\Delta K = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1 + m_2)v_{comb}^2$

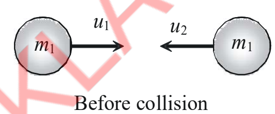
$\Delta K = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$ [By substituting the value of v_{comb}]

(2) **When the colliding bodies are moving in the opposite direction**

By the law of conservation of momentum

$m_1u_1 + m_2(-u_2) = (m_1 + m_2)v_{comb}$ (Taking left to right as positive)

$\therefore \quad v_{comb} = \dfrac{m_1u_1 - m_2u_2}{m_1 + m_2}$



when $m_1u_1 > m_2u_2$ then $v_{comb} > 0$ (positive)

i.e. the combined body will move along the direction of motion of mass m_1.

when $m_1u_1 < m_2u_2$ then $v_{comb} < 0$ (negative)

i.e. the combined body will move in a direction opposite to the motion of mass .

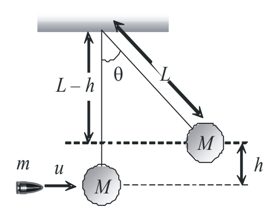
(3) **Loss in kinetic energy**

$\Delta K =$ Initial kinetic energy – Final kinetic energy $= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}(m_1 + m_2)v_{comb}^2\right)$

$= \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 + u_2)^2$

Collision Between Bullet and Vertically Suspended Block.

A bullet of mass m is fired horizontally with velocity u in block of mass M suspended by vertical thread. After the collision bullet gets embedded in block. Let the combined system raised upto height h and the string makes an angle θ with the vertical.



(1) **Velocity of system**

Let v be the velocity of the system (block + bullet) just after the collision.

$\text{Momentum}_{bullet} + \text{Momentum}_{block} = \text{Momentum}_{bullet\ and\ block\ system}$

$mu + 0 = (m + M)v$

$$\therefore \quad v = \frac{mu}{(m+M)} \qquad \ldots\ldots(i)$$

(2) **Velocity of bullet :** Due to energy which remains in the bullet block system, just after the collision, the system (bullet + block) raises upto height *h*.

By the conservation of mechanical energy $\frac{1}{2}(m+M)v^2 = (m+M)gh \Rightarrow v = \sqrt{2gh}$

Now substituting this value in the equation (i) we get $\sqrt{2gh} = \frac{mu}{m+M}$

$$\therefore \quad u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$$

(3) **Loss in kinetic energy :** We know the formula for loss of kinetic energy in perfectly inelastic collision

$$\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 + u_2)^2$$

$$\therefore \quad \Delta K = \frac{1}{2}\frac{mM}{m+M}u^2 \qquad [\text{As } u_1 = u,\ u_2 = 0,\ m_1 = m \text{ and } m_2 = M]$$

(4) **Angle of string from the vertical**

From the expression of velocity of bullet $u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$ we can get $h = \frac{u^2}{2g}\left(\frac{m}{m+M}\right)^2$

From the figure

$$\cos\theta = \frac{L-h}{L} = 1 - \frac{h}{L}$$

$$= 1 - \frac{u^2}{2gL}\left(\frac{m}{m+M}\right)$$

$$\text{or} \quad \theta = \cos^{-1}\left[1 - \frac{1}{2gL}\left(\frac{mu}{m+M}\right)^2\right]$$

SOLVED EXAMPLE

Example 1. n small balls each of mass m impinge elastically each second on a surface with velocity u. The force experienced by the surface will be

(A) mnu (B) 2 mnu

(C) 4 mnu (D) $\frac{1}{2}mnu$

Solution. **(B)**

As the ball rebounds with same velocity therefore change in velocity = 2u and the mass colliding with the surface per second = nm

Force experienced by the surface $F = m\frac{dv}{dt}$ $\therefore$ F = 2 mnu.

Example 2. A block of mass 12kg moving at 20 cm/s collides with an identical stationary block. If the coefficient of restitution in 3/5 the loss in K.E during collision is -

Solution. The loss in K.E $\Delta E = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(1 - e^2)\,(u_1 - u_2)^2$

Here m_1 = 2kg, m_2 = 2kg, e = 3/5

u_1 = 20 cm/sec = 0.2 m/s, u_2 = 0

On substituting the values

$$\Delta E = \frac{1}{2}\left(\frac{12\times 12}{12+12}\right)\left(1-\frac{9}{25}\right)(0.2)^2 = \frac{1}{2}\times 6\times\frac{16}{25}\times\frac{2}{10}\times\frac{2}{10} \cong 7.7\times 10^{-2}\,J$$

Example 3. A ball is dropped from a height h on a stationary floor and rebounds several times until it stops. If the coefficient of restitution is e, then the total distance covered by the ball before it stops, would be

Solution. The height h_1 up to which the ball rises after the first rebound is given by

$h_1 = e^2 h$

After second rebound, $h_2 = e^4 h$

After rebounding n times, $h_n = (e^2)^n h$ Total distance described

$s = h + 2h_1 + 2h_2 + ... + 2h_n + ...$

$= h + 2e^2h + 2e^4h + ... + 2e^{2n}h + ...$ $= h + 2e^2h\,(1 + e^2 + e^4 + ... \infty)$

$$= h + \frac{2e^2h}{1-e^2} = \left(\frac{1+e^2}{1-e^2}\right)h$$

Example 4. A rifle man, who together with his rifle has a mass of 100 kg, stands on a smooth surface fires 10 shots horizontally. Each bullet has a mass 10 gm a muzzle velocity of 800 m/s. What velocity does rifle man acquire at the end of 10 shots.

Solution. Let m_1 and m_2 be the masses of bullet and the rifleman and v_1 and v_2 their respective velocities after the first shot. Initially the rifleman and bullet are at rest, therefore initial momentum of system = 0.

i.e. initial momentum = final momentum = $m_1v_1 + m_2v_2$

or $$v_2 = \frac{m_1v_1}{m_2} = -\frac{(10\times 10^{-3}\,kg)(800\,m/s)}{100\,kg}$$

= – 0.08 m/s

Velocity acquired after 10 shots = 10 v_2 = 10 × (–0.08)

= – 0.8 m/s

i.e., the velocity of rifle man is 0.8 m/s in a direction opposite to that of bullet.

Example 5. A bullet of mass 10 g travelling horizontally with a velocity 300 m/s strikes a block of wood of mass 290 g which rests on a rough horizontal floor. After impact the block and the bullet move together and come to rest when the block has travelled a distance of 15 m. The coefficient of friction between the block and the floor will be (Duration of impact is very short)

Solution. Let the mass of block and bullet be M and m respectively. If v is the velocity of bullet and V is the velocity of block with bullet embedded in it,

Now according to conservation of momentum,

$mv = (M + m) V$

$(10 \times 10^{-3}) (300/ = (290 \times 10^{-3} + 10 \times 10^{-3}) V$

or $V = 10$ mls

The kinetic energy just after impact is $1/2(M + m) V^2$, which is lost due to work done on it by the force of friction F. Since force of friction F = m(M+m)g and the work done is given by Fd, we have

$1/2 (M + m) V^2 = \mu (M + m) gd$

or $$m = \frac{1}{2}\frac{V^2}{gd} = \frac{1}{2}\times\frac{10^2}{(10)(15)} = \frac{1}{3}$$

NOTE- Here an external horizontal force due to friction is present however as it has been assumed that impact lasted for such a small interval of time that the block could not move appreciably no work was done by friction during impact. Here during impact the presence of friction cannot be ignored.

Example 6. A block of mass $m_1 = 150$ kg is at rest on a very long frictionless table, one end of which is terminated in a wall. Another block of mass m_2 is placed between the first block and the wall, and set in motion towards m_1 with constant speed u_2 Assuming that all collisions are completely elastic, find the value of m_2 for which both blocks move with the same velocity after m_2 has collided once with m_1 and once with the wall. (The wall has effectively infinite mass.)

Solution. Let after the collision,

v_1 = speed of mass m_1 towards left

v_2 = speed of mass m_2 towards right.

Hence, momentum before collision = momentum after collision

$m_2u_2 = m_1v_1 - m_2v_2$ .....(1)

The mass m_2 rebounds elastically from the wall and its velocity gets reversed after the collision with the wall.

According to the problem, the mass m_2 has the same speed as that of mass m1 after its collision with the wall i.e. $v_2 = v_1$. From eq.

$m_2u_2 = (m_1 - m_2) v_1$ .....(2)

Since the collision is elastic, then $\frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_1^2$

$m_2u_2^2 = (m_1 + m_2) v_1^2$ ......(3)

From eq. (2), $v_1 = \dfrac{m_2 u_2}{m_1 - m_2}$

Substituting this value of v_1 in eq. (3), we get

$$m_2 u_2^2 = \frac{(m_1 + m_2)(m_2 u_2)^2}{(m_1 - m_2)^2}$$

or $\quad (m_1 - m_2)^2 = (m_1 + m_2)\,(m_2)$

or $\quad m_1^2 + m_2^2 - 2m_1 m_2 = m_1 m_2 + m_2^2 \quad$ or $\quad m_1^2 = 3\, m_1 m_2\; m_1 = 3m_2$

or $\quad m_2 = \dfrac{m_1}{3} = \dfrac{150}{3} = 50.\text{kg}$

Example 7. A ball moving with a speed of 9 m/s strikes with an identical stationary ball such that after the collision the direction of each ball makes an angle of 30° with the original line of motion. Find the speeds of the two balls after the collision. Is the kinetic energy conserved in this collision process?

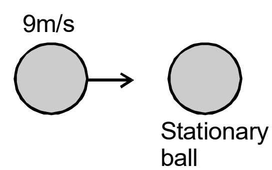
Solution. Initial momentum of the balls = $m \times 9 + m \times 0 = 9\,m$ ...(1)

where m is the mass of each ball.

Let after collision their velocities are v_1 and v_2 respectively.

Final momentum of the balls after collision along the same line = $mv_1 \cos 30 + mv_2 \cos 30$

$$= \frac{mv_1\sqrt{3}}{2} + \frac{mv_2\sqrt{3}}{2} \qquad ...(2)$$



According to law of conservation of momentum

$$9m = \frac{mv_1\sqrt{3}}{2} + \frac{mv_2\sqrt{3}}{2}$$

$$\frac{9 \times 2}{\sqrt{3}} = v_1 + v_2 \qquad ...(3)$$

(A) Before collision

The initial momentum of the balls along perpendicular direction = 0.

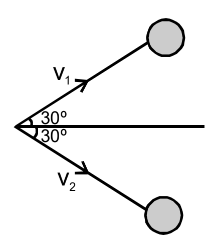
Final momentum of balls along the perpendicular direction

$$= mv_1 \sin 30 - mv_2 \sin 30 = \frac{m}{2}(v_1 - v_2)$$

Again by the law of conservation of momentum

$(m/2)(v_1 - v_2) = 0$

$\therefore \quad (v_1 - v_2) = 0 \qquad$ .....(4)



Solving equations (3) and (4), we have (B) After collision

$v_1 = 3\sqrt{3}$ m/s and $v_2 = 3\sqrt{3}$ m/s

According to law of conservation of energy.

Energy before collision = Energy after collision

$$\frac{1}{2}mu_1^2 + \frac{1}{2}m\,u_2^2 = \frac{1}{2}m\,v_1^2 + \frac{1}{2}m\,v_2^2 \quad \Rightarrow \quad \frac{1}{2}m(9)^2 + 0 = \frac{1}{2}m(3\sqrt{3})^2 + \frac{1}{2}m(3\sqrt{3})^2$$

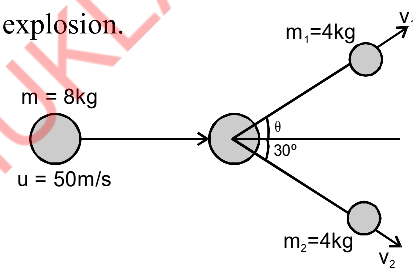
$$\frac{81\,m}{2} = \frac{54\,m}{2} \quad \text{L.H.S.} \neq \text{R.H.S.}$$

i.e., energy is not conserved in this collision or this is a case of inelastic collision.

Example 8. A sphere of mass 8 kg moving at constant speed 50 m/s, contains a compressed light spring with strain energy 15,000 joule. At a given instant, the spring breaks and causes the sphere to explode into tow pieces of equal masses. If one piece flies off at 30° to the original velocity of the sphere, find the direction of motion of the other piece and magnitudes of the velocities of the two pieces. Assume the energy of the compressed spring is completely imparted to the two pieces all kinetic energy.

Solution. The situation is shown in fig.

Let v_1 and v_2 be the velocities of two pieces after explosion.



Applying the law of conservation of energy,

we have

$$\frac{1}{2}(8)(50)^2 + 15000 = \frac{1}{2}(4)v_1^2 + \frac{1}{2}(4)v_2^2$$

or $\quad 25000 = 2(v_1^2 + v_2^2) \qquad$ ....(1)

Applying the law of conservation of momentum along x-axis and y-axis respectively, we get

$8(5) = 4\,v_1 \cos\theta + v_2 \cos 30° \qquad$ ....(2)

and $\quad 0 = 4v_1 \sin\theta = 4\,v_2 \sin 30° = 2v_2 \qquad$ ....(3)

or $\quad \sin\theta = \dfrac{v_2}{2v_1} \qquad$ ....(4)

From eq. (2)

$100 = v_1 \cos\theta + v_2 \cos 30$

or $\quad 100 = v_1\sqrt{\left(1 - \dfrac{v_2^2}{4v_1^2}\right)} + v_2\left(\dfrac{\sqrt{3}}{2}\right)$ or $100 = v_1\dfrac{\sqrt{4v_1^2 - v_2^2}}{2} + \dfrac{v_2\sqrt{3}}{2} \qquad$ ....(5)

Solving equations (1) and (5) for v_1 and v_2 we get

$v_1 = 51.56$ m/s and $v_2 = 99.2$ m/s

Now $\sin\theta = \dfrac{99.2}{2 \times 51.56}$ Solving we get $\theta = 74°8$

EXERCISE

1. A particle of mass m moving with horizontal speed 6 m/sec. If m<<M then for one dimensional elastic collision, the speed of lighter particle after collision will be

(A) 2 m/sec in original direction
(B) 2 m/sec opposite to the original direction
(C) 4 m/sec opposite to the original direction
(D) 4 m/sec in original direction

2. A body of mass m moving with velocity v makes a head-on collision with another body of mass 2m which is initially at rest. The loss of kinetic energy of the colliding body (mass m) is

(A) $\frac{1}{2}$ of its initial kinetic energy
(B) $\frac{1}{9}$ of its initial kinetic energy
(C) $\frac{8}{9}$ of its initial kinetic energy
(D) $\frac{1}{4}$ of its initial kinetic energy

3. A ball of mass m moving with velocity V, makes a head on elastic collision with a ball of the same mass moving with velocity 2V towards it. Taking direction of V as positive velocities of the two balls after collision are

(A) – V and 2V
(B) 2V and – V
(C) V and – 2V
(D) – 2V and V

4. Consider the following statements

Assertion (A) : In an elastic collision of two billiard balls, the total kinetic energy is conserved during the short time of collision of the balls (i.e., when they are in contact)

Reason (R) : Energy spent against friction does not follow the law of conservation of energy of these statements

(A) Both A and R are true and the R is a correct explanation of A
(B) Both A and R are true but the R is not a correct explanation of the A
(C) A is true but the R is false
(D) Both A and R are false

5. A big ball of mass M, moving with velocity u strikes a small ball of mass m, which is at rest. Finally small ball attains velocity u and big ball v. Then what is the value of v

(A) $\frac{M-m}{M+m}u$
(B) $\frac{m}{M+m}u$
(C) $\frac{2m}{M+m}u$
(D) $\frac{M}{M+m}u$

6. A car of mass 400 kg and travelling at 72 kmph crashes into a truck of mass 4000 kg and travelling at 9 kmph, in the same direction. The car bounces back at a speed of 18 kmph. The speed of the truck after the impact is

(A) 9 kmph
(B) 18 kmph
(C) 27 kmph
(D) 36 kmph

7. A smooth sphere of mass M moving with velocity u directly collides elastically with another sphere of mass m at rest. After collision their final velocities are V and v respectively. The value of v is

(A) $\frac{2uM}{m}$ (B) $\frac{2um}{M}$

(C) $\frac{2u}{1+\frac{m}{M}}$ (D) $\frac{2u}{1+\frac{M}{m}}$

7. A sphere of mass 0.1 kg is attached to a cord of 1m length. Starting from the height of its point of suspension this sphere hits a block of same mass at rest on a frictionless table, If the impact is elastic, then the kinetic energy of the block after the collision is

(A) 1 J (B) 10 J

(C) 0.1 J (D) 0.5 J

8. A moving body with a mass m_1 strikes a stationary body of mass m_2. The masses m_1 and m_2 should be in the ratio $\frac{m_1}{m_2}$ so as to decrease the velocity of the first body 1.5 times assuming a perfectly elastic impact. Then the ratio is

(A) 1/25 (B) 1/5

(C) 5 (D) 25

9. A moving mass of 8 kg collides elastically with a stationary mass of 2 kg. If E be the initial kinetic energy of the mass, the kinetic energy left with it after collision will be

(A) 0.80 E (B) 0.64 E

(C) 0.36 E (D) 0.08 E

10. A neutron travelling with a velocity v and K.E. E collides perfectly elastically head on with the nucleus of an atom of mass number A at rest. The fraction of total energy retained by neutron is

(A) $\left(\frac{A-1}{A+1}\right)^2$ (B) $\left(\frac{A+1}{A-1}\right)^2$

(C) $\left(\frac{A-1}{A}\right)^2$ (D) $\left(\frac{A+1}{A}\right)^2$

11. Which one of the following statement does not hold well when two balls of masses m_1 and m_2 undergo elastic collision

(A) When $m_1 < m_2$ and at rest, there will be maximum transfer of momentum

(B) When and m_2 at rest, after collision the ball of mass m_2 moves with four times the velocity of m_1

(C) When and at rest, there will be maximum transfer of kinetic energy

(D) When collision is oblique and at rest with after collision the balls move in opposite directions

12. A ball moving with velocity of 9 m/s collides with another similar stationary ball. After the collision both the balls move in directions making an angle of 30^{o} with the initial direction. After the collision their speed will be

(A) 2.6 m/s (B) 5.2 m/s

(C) 0.52 m/s (D) 52 m/s

13. A ball B_1 of mass M moving northwards with velocity v collides elastically with another ball B_2 of same mass but moving eastwards with the same velocity v. Which of the following statements will be true

(A) Comes to rest but moves with velocity

(B) Moves with velocity but comes to rest

(C) Both move with velocity in north east direction

(D) Moves eastwards and moves north wards

14. A body of mass $40kg$ having velocity 4 m/s collides with another body of mass 60 kg having velocity 2 m/s. If the collision is inelastic, then loss in kinetic energy will be

(A) 440 J (B) 392 J

(C) 48 J (D) 144 J

15. One sphere collides with another sphere of same mass at rest inelastically. If the value of coefficient of restitution is $\frac{1}{2}$, the ratio of their speeds after collision shall be

(A) 1 : 2 (B) 2 : 1

(C) 1 : 3 (D) 3 : 1

16. The ratio of masses of two balls is 2 : 1 and before collision the ratio of their velocities is 1 : 2 in mutually opposite direction. After collision each ball moves in an opposite direction to its initial direction. If e = (5/6), the ratio of speed of each ball before and after collision would be

(A) (5/6) times

(B) Equal

(C) Not related

(D) Double for the first ball and half for the second ball

17. The change of momentum in each ball of mass 60 gm, moving in opposite directions with speeds 4 m/s collide and rebound with the same speed, is

(A) 0.98 kg–m/s (B) 0.73 kg–m/s

(C) 0.48 kg–m/s (D) 0.22 kg–m/s

18. A body falling from a height of 20m rebounds from hard floor. If it loses 20% energy in the impact, then coefficient of restitution is

(A) 0.89 (B) 0.56

(C) 0.23 (D) 0.18

19. A rubber ball is dropped from a height of 5 m on a planet where the acceleration due to gravity is not known. On bouncing, it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of

(A) 16/25 (B) 2/5

(C) 3/5 (D) 9/25

20. Which of the following is not a perfectly inelastic collision

(A) Striking of two glass balls (B) A bullet striking a bag of sand

(C) An electron captured by a proton (D) A man jumping onto a moving cart

21. A metal ball of mass 2 kg moving with a velocity of 36 km/h has an head-on collision with a stationary ball of mass 3 kg. If after the collision, the two balls move together, the loss in kinetic energy due to collision is

(A) 40 J (B) 60 J

(C) 100 J (D) 140 J

22. A mass of 20 kg moving with a speed of 10 m/s collides with another stationary mass of 5 kg. As a result of the collision, the two masses stick together. The kinetic energy of the composite mass will be

(A) 600 J (B) 800 J

(C) 1000 J (D) 1200 J

23. A neutron having mass of $1.67 \times 10^{-27} kg$ and moving at 10^8 m/s collides with a deuteron at rest and sticks to it. If the mass of the deuteron is 3.34×10^{-27}; kg the speed of the combination is

(A) 2.56×10^3 m/s (B) 2.98×10^5 m/s

(C) 3.33×10^7 m/s (D) 5.01×10^9 m/s

Important Competition Tips

1. *The area under the force-displacement graph is equal to the work done.*
2. *Work done by a centripetal force is always zero.*
3. *Energy is a promise of work to be done in future. It is the stored ability to do work.*
4. *When work is done on a body, its kinetic or potential energy increases.*
5. *When the work is done by the body, its potential or kinetic energy decreases.*
6. *According to the work energy theorem, the work done is equal to the change in energy. That is* $W = \Delta E$.
7. *Potential energy of a system increases when a conservative force does work on it.*
8. *The kinetic energy of a body is always positive.*
9. *When the momentum of a body increases by a factor n, then its kinetic energy is increased by factor* n^2.
10. *The total energy (including mass energy) of the universe remains constant.*
11. *One form of energy can be changed into other form according to the law of conservation of energy. That is amount of energy lost of one form should be equal to energy or energies produced of other forms.*
12. *Kinetic energy can change into potential energy and vice versa.*

    When a body falls, potential energy is converted into kinetic energy.
13. *Work done by conservative forces is indepeudeul of the path followed but depends only on initial and final position.*

    e.g: gravitational and electric field.
14. *Work done by non - conservative force is path dependent e.g: friction, viscose force*
15. $e = \dfrac{\vec{v}_2 - \vec{v}_1}{\vec{u}_1 - \vec{u}_2}$ *is called coefficient of restitution. Its value is 1 for elastic collisions. It is less than 1 for inelastic collisions and zero for perfectly inelastic collision.*
16. *During collision, velocity of the colliding bodies changes.*
17. *Linear momentum is conserved in all types of collisions.*
18. *Perfectly elastic collision is a rare physical phenomenon.*
19. *Collisions between two ivory or steel or glass balls are nearly elastic.*
20. *The force of interaction in an inelastic collision is non-conservative in nature.*
21. *In inelastic collision, the kinetic energy is converted into heat energy, sound energy, light energy etc.*
22. *In head on collisions, the colliding bodies move along the same straight line before and after collision.*
23. *Head on collisions are also called one dimensional collisions.*
24. *In the oblique collisions the colliding bodies move at certain angles before and/or after the collisions.*
25. *The oblique collisions are two dimensional collisions.*
26. *When a heavy body collides head-on elastically with a lighter body, then the lighter body begins to move with a velocity nearly double the velocity of the heavier body.*
27. *When a light body collides with a heavy body, the lighter body returns almost with the same speed.*
28. *If a light and a heavy body have equal momenta, then lighter body has greater kinetic energy.*

29. *Suppose, a body is dropped form a height h_0 and it strikes the ground with velocity v_0. After the (inelastic) collision let it rise to a height h_1. If v_1 be the velocity with which the body rebounds, then*

$$e = \frac{v_1}{v_0} = \left[\frac{2gh_1}{2gh_0}\right]^{1/2} = \left[\frac{h_1}{h_0}\right]^{1/2}$$

30. *If after n collisions with the ground, the velocity is v_n and the height to which it rises be h_n, then*

$$e^n = \frac{v_n}{v_0} = \left[\frac{h_n}{h_0}\right]^{1/2}$$

31. $P = \vec{F}.\vec{v} = Fv\cos\theta$ *where $\vec{v}$ is the velocity of the body and θ is the angle between $\vec{F}$ and $\vec{v}$*

32. *Area under the $F-v$ graph is equal to the power dissipated.*

33. *Power dissipated by a conservative force (gravitation, electric force etc.) does not depend on the path followed. It depends on the initial and final positions of the body. That is $\oint dP = 0$.*

34. *Power dissipated against friction depends on the path followed. That is $\oint dP \neq 0$.*

35. *Power is also measured in horse power (hp). It is the fps unit of power. 1 hp = 746 W.*

36. *An engine pulls a train of mass m with constant velocity. If the rails are on a plane surface and there is no friction, the power dissipated by the engine is zero.*

37. *In the above case if the coefficient of friction for the rail is μ, the power of the engine is $P = \mu mgv$.*

38. *In the above case if the engine pulls on a smooth track on an inclined plane (inclination θ), then its power $P = (mg\sin\theta)v$.*

39. *In the above case if the engine pulls upwards on a rough inclined plane having coefficient of friction μ, then power of the engine is $P = (\mu\cos\theta + \sin\theta)mg\,v$.*

40. *If the engine pulls down on the inclined plane then power of the engine is $P = (\mu\cos\theta - \sin\theta)mg\,v$.*